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NAL – 1

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1. Truth-values in IL – 1.

   There is ideal experience K in IL – 1, using the reasoning rules in IL – 1 (mainly the deduction rule), we can get the transitive closure K^*. Here, if we say an inheritance, say $S \to P$ is true, when $S \to P \in K^*$.

   For example, if we have $K = \{A \to B, B \to C\}$, we get $K^* = \{A \to B, B \to C, A \to C\}$, and so $A \to C$ is true, and $A \to D$ is false.

2. Based on the meaning of an inheritance relation, if $S \to P$ is true, then we usually say "S is a kind of P", or "S is a specification of P", or "P is a generalization of S". But if we want to discuss to what extent an inheritance is true, we assume the truth-value definition of IL – 1 is not valid anymore, since between true and false, we can have "kind of true" and "kind of false", which makes the binary truth-value a continuous one. The reason why we introduce a continuous truth is in our daily life, we usually don't use (or cannot have) such axioms. Say "it is going to rain", what is the truth? No matter what it is, it cannot be an absolute "true" or "false", but something in the middle.

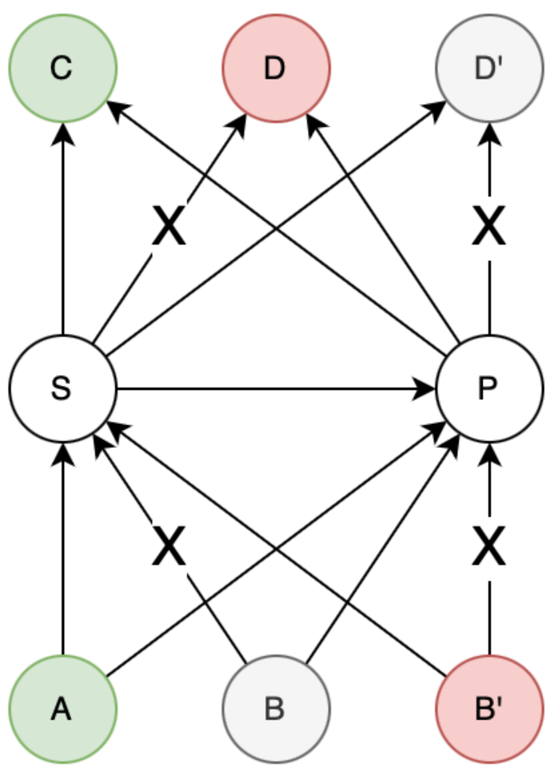
   To Talk about continuous truth-values, we may start with seeing how to get them from set theory. If we interpret S, P as sets and say $S = \{A, B, C\}, P = \{B, C, D\}$, then for B, C, they can be used for supporting evidence for "S is a kind of P" (note that I did not use $S \subseteq P$, since I am NOT talking about set theory, but use it to explain inheritance) (since B, C are both in S and P). But for A, since $A \in S$, but $A \notin P$, hence A is negative evidence for "S is a kind of P". For D, it is not useful in judging whether "S is a kind of P" is true or false. Assume S is a set of birds, and P is a set of flying beings. Then B, C are birds and flying beings, and they are positive evidence; A is a bird but not a flying being (penguin), and it is negative evidence; D is not a bird but a flying being (airplane), and it will not contradict with "bird is a kind of flying being". Therefore, in discussing whether an inheritance is true, we will need evidence, which can be separated in 3 groups: 1) positive evidence, 2) negative evidence, 3) unrelated evidence (like D above).

3. From the above example, it seems that we can use set theory (subset relationships) to discuss continuous truth-values, but we will not do it, since set theory is purely for extensional information (specifications), and intentional evidence (generalizations) is also very important in deciding the truth-value.

   For example, for an inheritance "brain → computer", which is usually not true, but we can still find evidence "computer → can_store_knowledge" and "brain → can_store_knowledge" to say it is kind of true.

   Or we know cats and dogs are kind of similar (since they are popular pets, intentional), but if we find a set for cat and another set for dog, we can hardly find "cat is a kind of dog", "dog is a kind of cat" true using set theory (since it is impossible for something being cat and dog at the same time, extensional), and say nothing to their similarity.

   Therefore, in NAL, we use the following schema of evidence. Among them, green ones are positive, red ones are negative, and gray ones are unrelated.



For any concept, we use S^E to represent its extension (specification), $\forall x, x \in S^E \coloneqq x \to S$. And S^I to represent its intention (generalization), $\forall x, x \in S^I \coloneqq S \to x$. Then we say the positive evidence for $S \to P$ is $(S^E \cap P^E) \cup (S^I \cap P^I)$; the extensional negative evidence is $S^E - P^E$; the intentional negative evidence is $P^I - S^I$. We say the (relative) number of positive evidence is w^+, negative evidence is w^-. Then the frequency of $S \to P$ is $f = w^+/(w^+ + w^-) \coloneqq w^+/w$.

4. Assume there is 1 positive and negative evidence for $S \to P$, and so its frequency if 0.5. If for some reason (inputting new information or getting results from reasoning), new evidence is added to $S \to P$. If this is positive evidence, then its frequency becomes 0.66, and 0.33 if it is negative. But I have no idea whether this new evidence is positive or negative, since it will be added in the future. For the above case, if you assume $f > 0.5$ to be "kind of true" and $f < 0.5$ to be "kind of false", then that inheritance is <u>not stable</u>. Since one unknown case may completely change your assessment to it. But if there are 10 positive evidence and 10 negatives at the beginning, <u>in dealing with one unknown case</u>, the upper bound (u) of its frequency can be 0.52, and the lower bound (l) will be 0.47, which is more stable.

   That is to say, except for the proportion of positive evidence, we also need to consider the amount of total evidence. Therefore, we introduced confidence $c = w/(w + k)$, in which k stands for "<u>in dealing with k unknown cases</u>".

   As a result, in NAL, we have 3 exchangeable representations of truth, 1) w^+, w^-, 2) f, c, 3) u, l. They are theoretically equivalent, but we may switch among them in the following discussion to make the explanation natural.

5. From here, we may receive some questions, "in discussing the continuous truth-value of inheritance, we used positive evidence and negative evidence, which are Boolean", so that the continuous truth-value is built upon Boolean truth-values, and so it is not necessary. This question can also be asked in talking about the truth-value using set theory, since though we agree $S \to P$ to some extent, it is based on the members of set S, P, and membership relationship is also Boolean.

   But in NAL, "whether this is positive/negative evidence" is also to some extent. Assume we are talking "bird is a kind of white being", then as human we know swan can be used to support it. But we also know not all swans are white. In this case "swan" as positive evidence, is not 100% positive. And that is why we <u>don't design w^+, w^- as sets containing evidence (though in the following discussion to make examples simpler, I will still use it if no confusions will be made)</u>, but just numbers representing the relative amount. Otherwise, we can just have 100% positive evidence, and 100% negative evidence, which is not flexible.

6. Revision rule

   Assume there is an inheritance $S \to P$, and we have evidence from $w^+ = \{A\}, w = \{A, B, C\}$ (here you may think $w^+ = 1, w = 3$, but I make it specifically where these 1 and 3 are from). And in another situation, I got the same sentence, $S \to P$, but it has evidence $w^+ = \{X\}, w = \{X, Y, Z\}$. Then we know we can combine these two sets of evidential and gain $S \to P$ more confidence.

   Therefore, for the same inheritance, if it has two truth-values from w_1^+, w_1 and w_2^+, w_2, we can combine them as $w^+ = w_1^+ + w_2^+, w = w_1 + w_2$ by the revision rule.

   Note that by using the revision rule, the premises are not canceled, you can still use the old premise to do other reasoning.

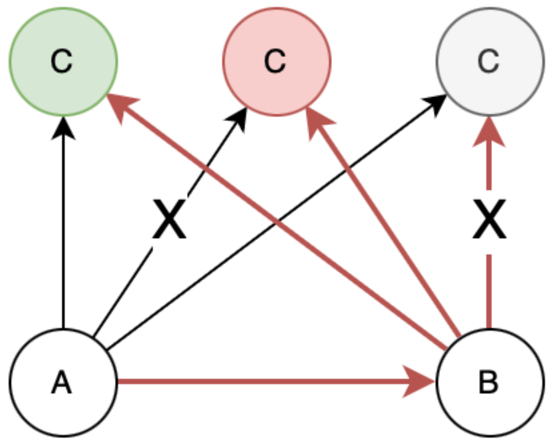
7. Choice rule

   But there is a problem here, since w^+, w^- are numbers, we don't know where these numbers are from, if $w_1^+ = \{A\}, w_1 = \{A, B, C\}, w_2^+ = \{A\}, w_2 = \{A, Y, Z\}$, combining them may use the same evidence twice. Therefore, we define <u>evidential base</u> to avoid such issues. An inheritance has evidential base just including itself at the beginning, and all derived inheritances will have the evidential base as the union of the premises. If the intersection of the two premises' evidential bases is not empty, a reasoning cannot be carried out. This results in some "look the same" inheritances with different truth-values and cannot be revised. In this case, to refer to the inheritance, we will use the choice rule to pick the one with the highest confidence.

8. Deduction rule

   In NAL, the deduction is represented as $B \to C < f_1, c_1 >, A \to B < f_2, c_2 > \vdash A \to C < f, c >$, here are two problems: 1) to make it natural, we will use $A \to B, B \to C$ as the premise, but why we choose $B \to C, A \to B$; 2) how the truth of $A \to C$ is defined.

To answer the first question, we need to make it clear what is different from positive/negative evidence against unrelated evidence. We may see when $B \to C$ is not true, we can only talk about unrelated evidence, and so to use this rule, we must make sure $B \to C$ is "kind of true", so we put it in the first place.

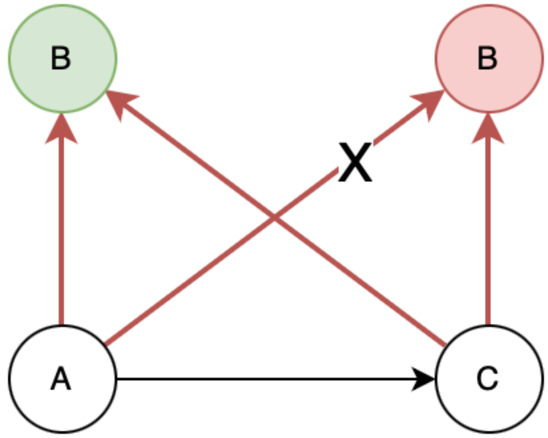


To answer the second question, we need to consider the same rule in IL – 1, "$A \to C$ is true when $B \to C, A \to B$ are both true", therefore, we say $f = f_1 f_2$. For confidence, only positive evidence is used (since we cannot separate intentional and extensional negative evidence), so $c = f_1 c_1 f_2 c_2$.

9. Abduction rule

   Write the deduction rule first, $B \to C, A \to B \vdash A \to C$, then exchange the conclusion and the second premise, we get $B \to C, A \to C \vdash A \to B$, and then rename B, C with each other, and we get the abduction rule $C \to B < f_1, c_1 >, A \to B < f_2, c_2 > \vdash A \to C < f, c >$.

   But the analyzing of its truth is different from above, since here these two premises make B intentional positive/negative evidence.

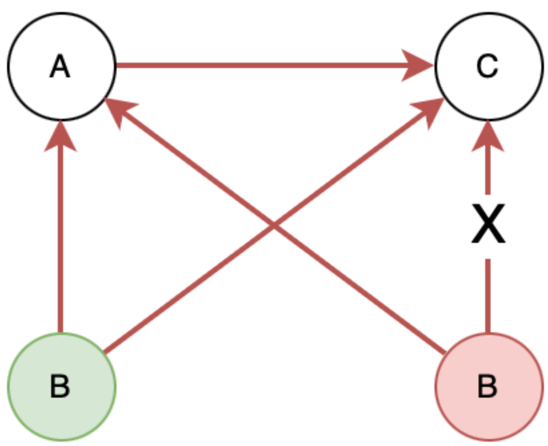


Therefore, $w^+ = f_1c_1f_2c_2, w^- = f_1c_1(1-f_2)c_2$. For similar reasons, we put $C \rightarrow B$ in the first place.

Note that, $c = w/(w+k) = f_1c_1c_2/(f_1c_1c_2 + k)$, and $f_1c_1c_2 \leq 1, k \geq 1$, so for this rule, the conclusion will always have confidence smaller than 0.5 (unreliable), so it is called a "weak" rule. Such rules are usually not allowed in many mathematical logic reasoning systems, but in NAL, we can still make it since it ACTUALLY uses a part of the evidence.

10. Induction rule

Like the abduction rule, but we exchange the conclusion with the first premise then rename it, and we will get $B \rightarrow C < f_1, c_1 >, B \rightarrow A < f_2, c_2 > \vdash A \rightarrow C < f, c >$.



If it is from the same reasoning like in deduction, we should put $B \rightarrow A$ in the first place, but to keep the way how it is derived from deduction (like induction), we will put it in the second place. But we cannot use this rule when $B \rightarrow A$ is "kind of false".

11. Conversion rule

Consider using abduction rule in this way, by replacing A with B, we have $C \to B, B \to B \vdash B \to C$, and since $B \to B$ is a tautology, its truth is $f = 1, c = 1$, and we can ignore it. Then this rule becomes $C \to B \vdash B \to C$.

12. Exemplification rule

This can be viewed as 1) deduction first, 2) conversion, but it is not. Since if so, 1) we should have the rule like $B \to C, A \to B \vdash C \to A$, but it is $A \to B, B \to C \vdash C \to A$. And 2) its truth should be $f = 1, c = f_1^2 f_2^2 c_1 c_2 / (f_1^2 f_2^2 c_1 c_2 + k)$, but its definition is $w^+ = f_1 f_2 c_1 c_2, w^- = 0$, so $f = 1, c = f_1 f_2 c_1 c_2 / (f_1 f_2 c_1 c_2 + k)$.

Therefore, we'd better take it as a stand-alone reasoning rule, considering the conversion rule, it will just use the positive evidence of premises, and by deduction rule, the positive evidence should be $f_1 f_2 c_1 c_2$.