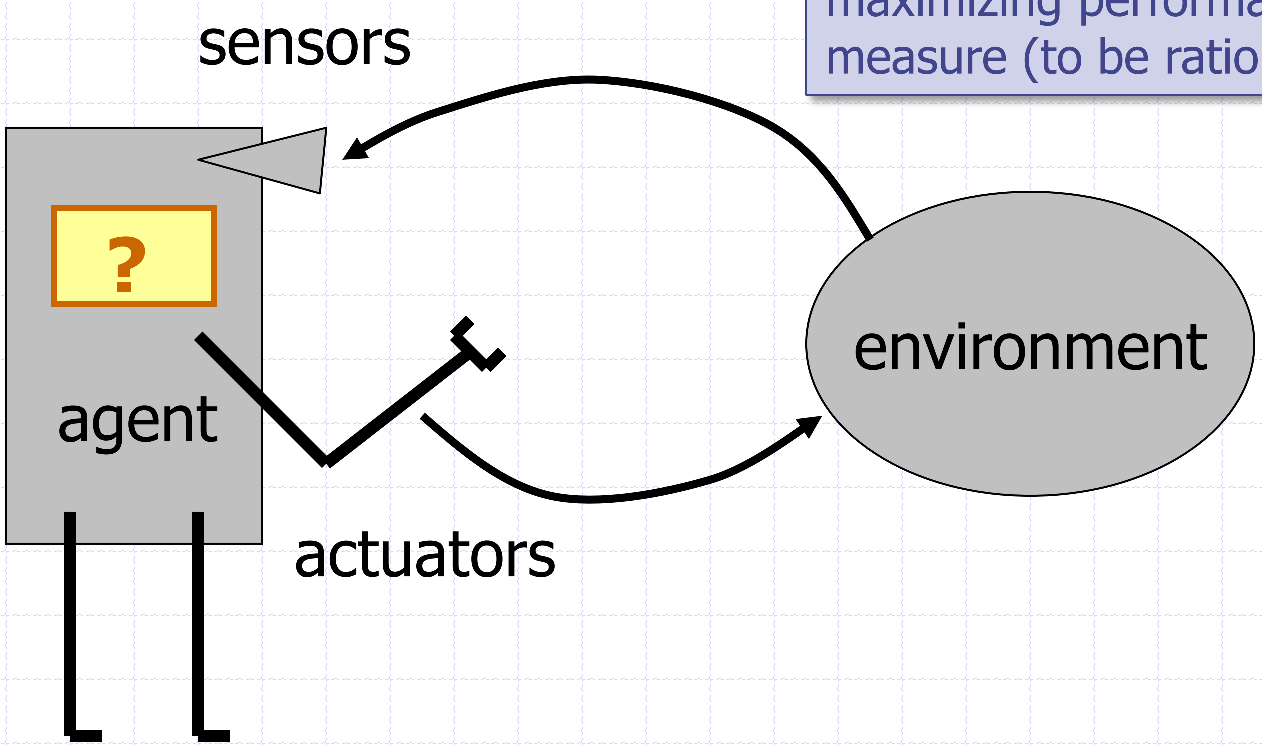
Search Problems

Russell and Norvig:
Chap. 3, Sect. 3.1 – 3.2

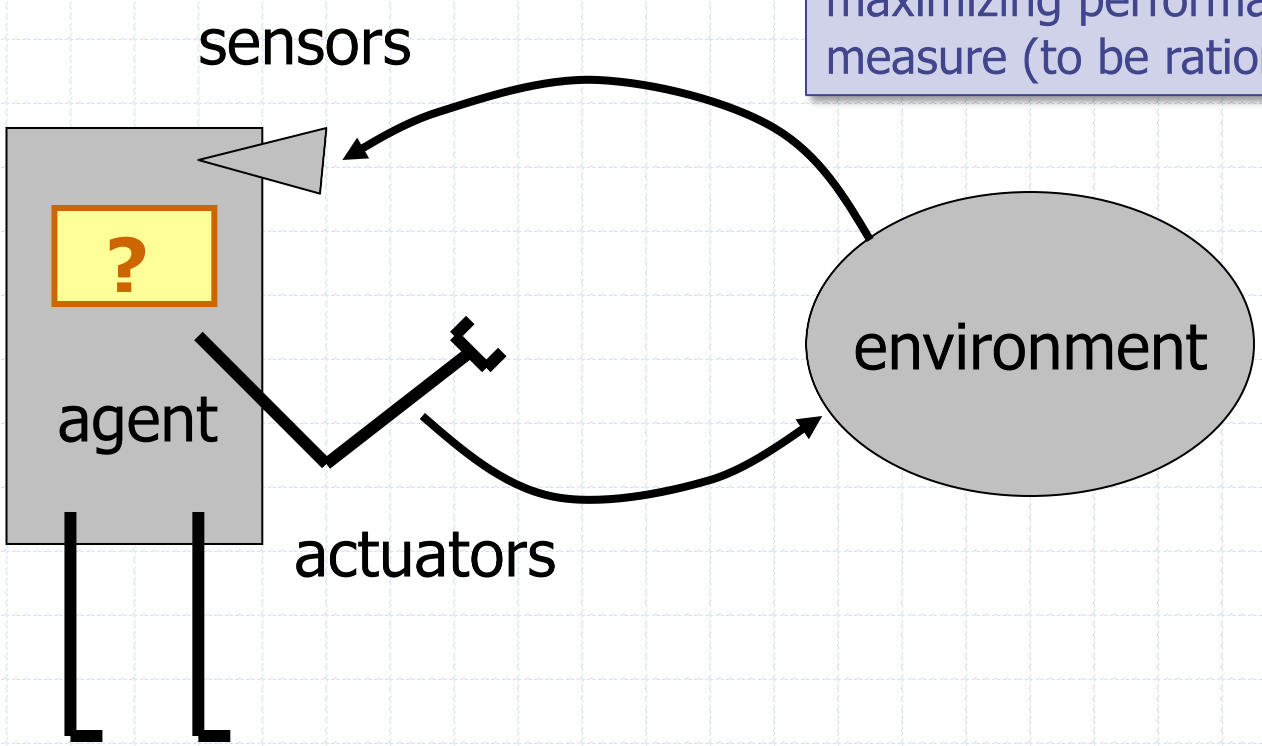
Slides by Jean-Claude Latombe, from an introductory AI course given at Stanford University (used with permission).

Goal-Based Agent

Goal: One way towards maximizing performance measure (to be rational)



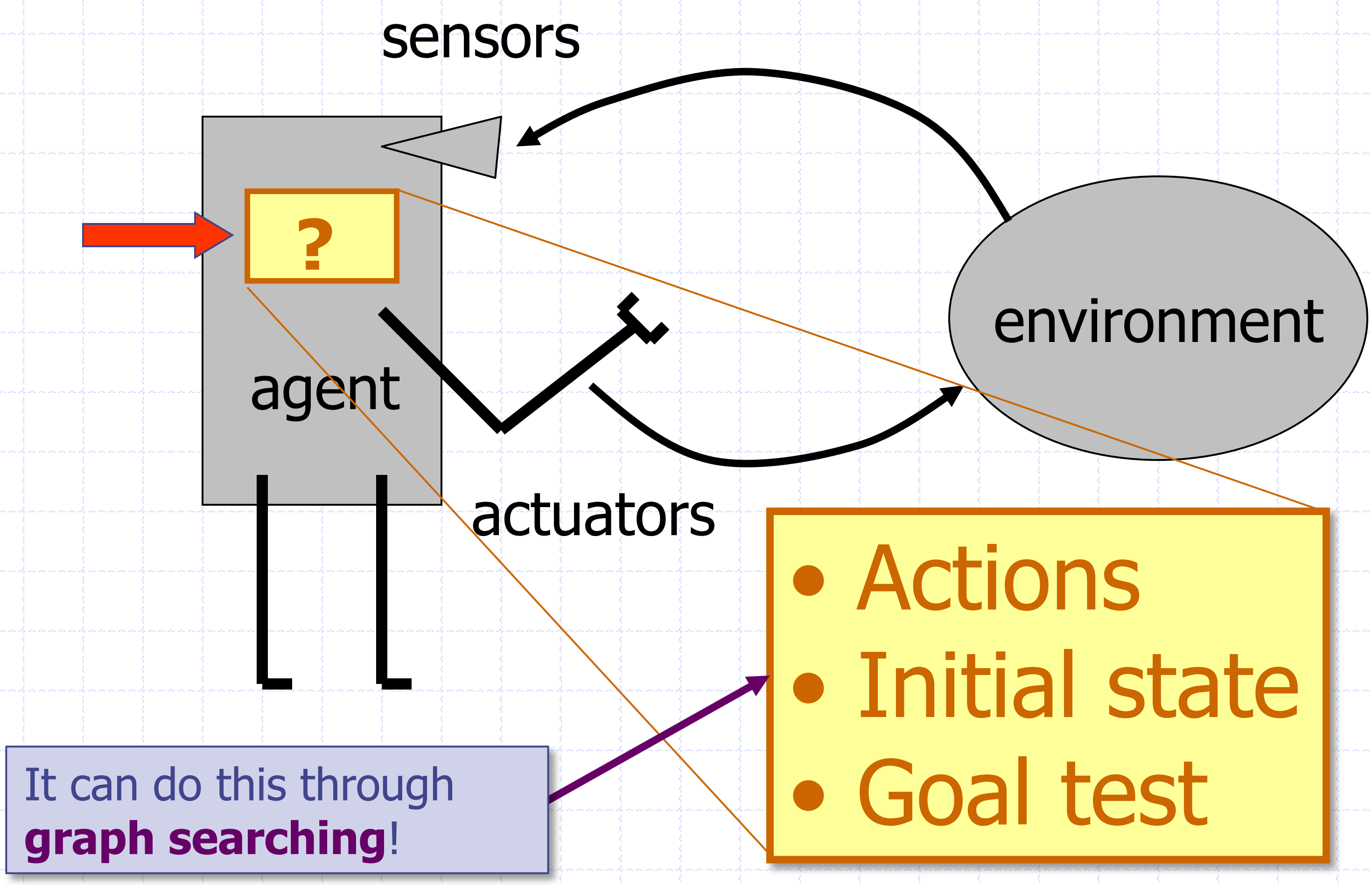
Goal-Based Agent



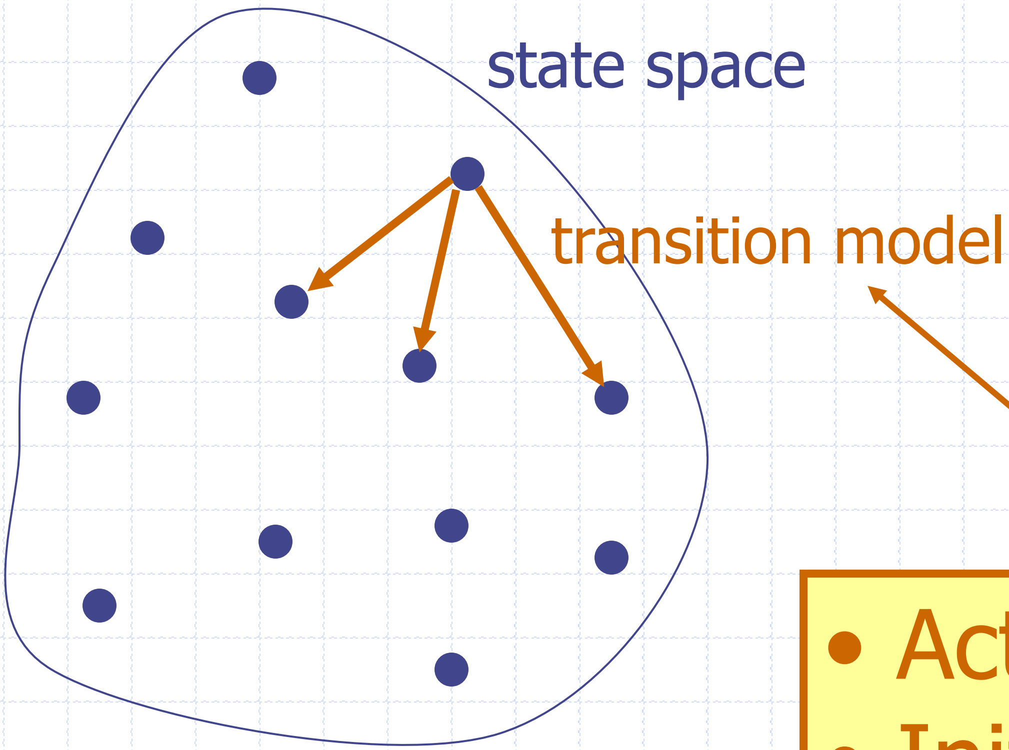
Goal: One way towards maximizing performance measure (to be rational)

Can it find a **sequence of actions** achieving its goals, when no single action will do?

Problem-Solving Agent

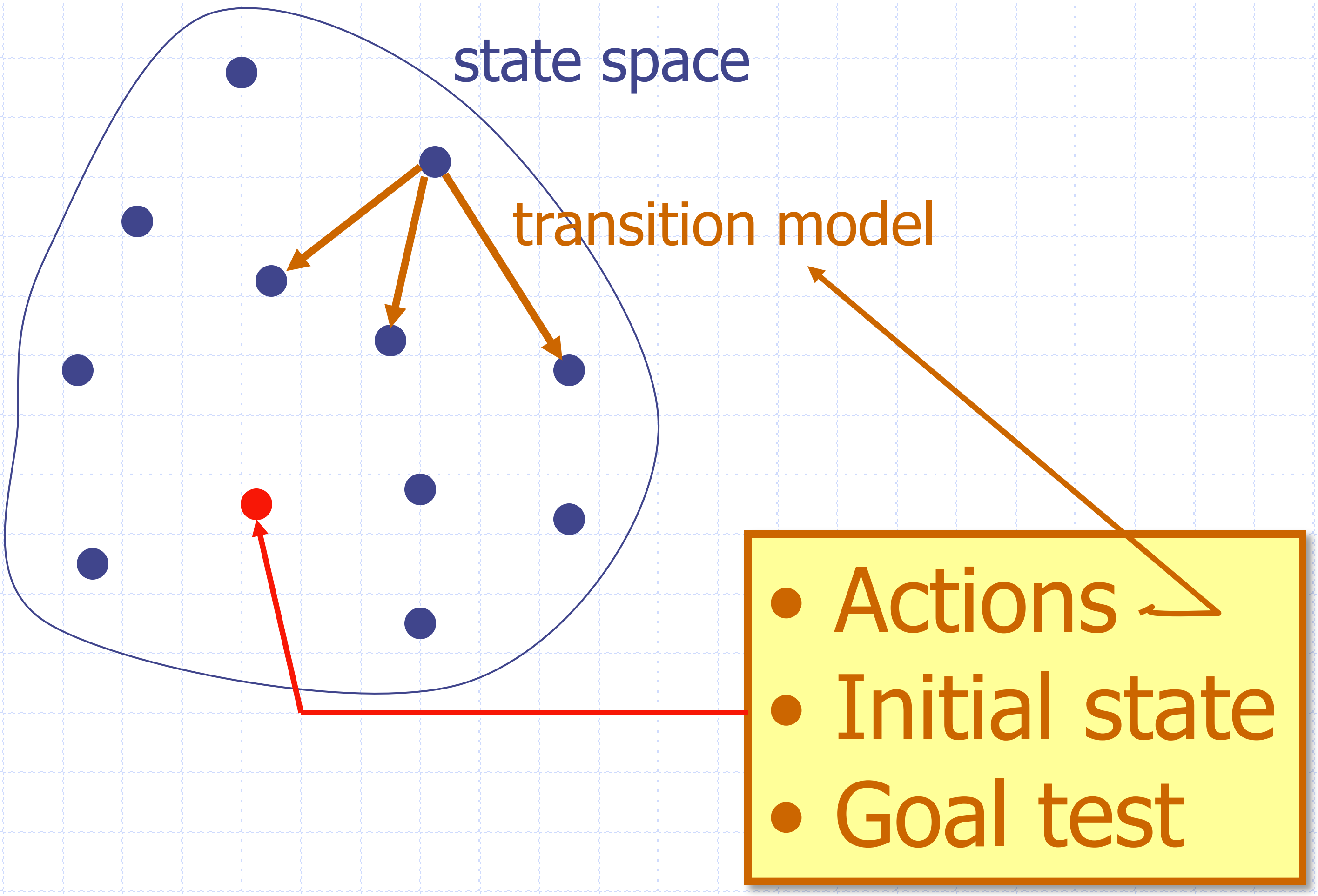


Problem as a Search Problem

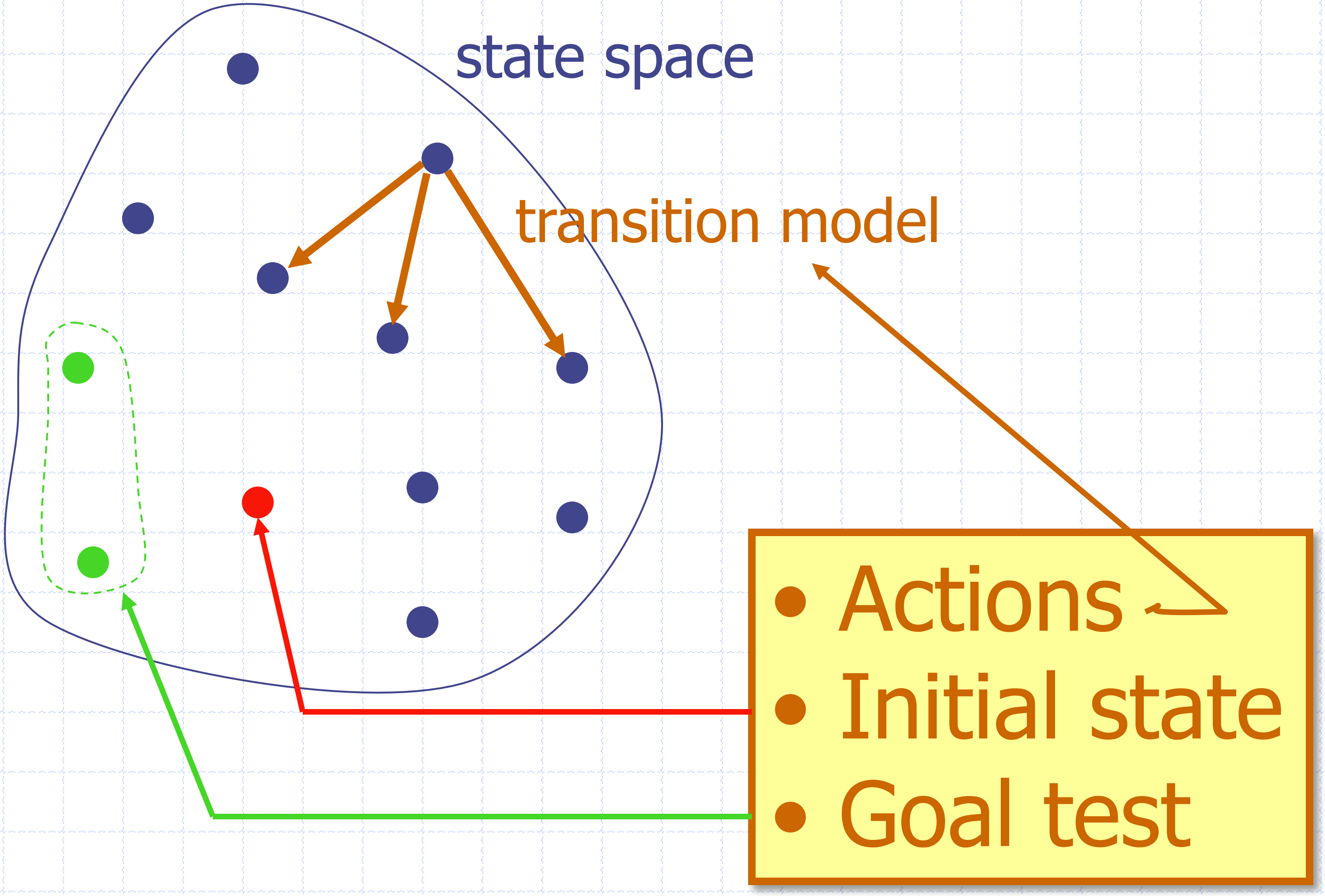


- Actions
- Initial state
- Goal test

Initial State



Goal Test



state space
transition model
• Actions
• Initial state
• Goal test

Example: 8-puzzle

8	2	
3	4	7
5	1	6

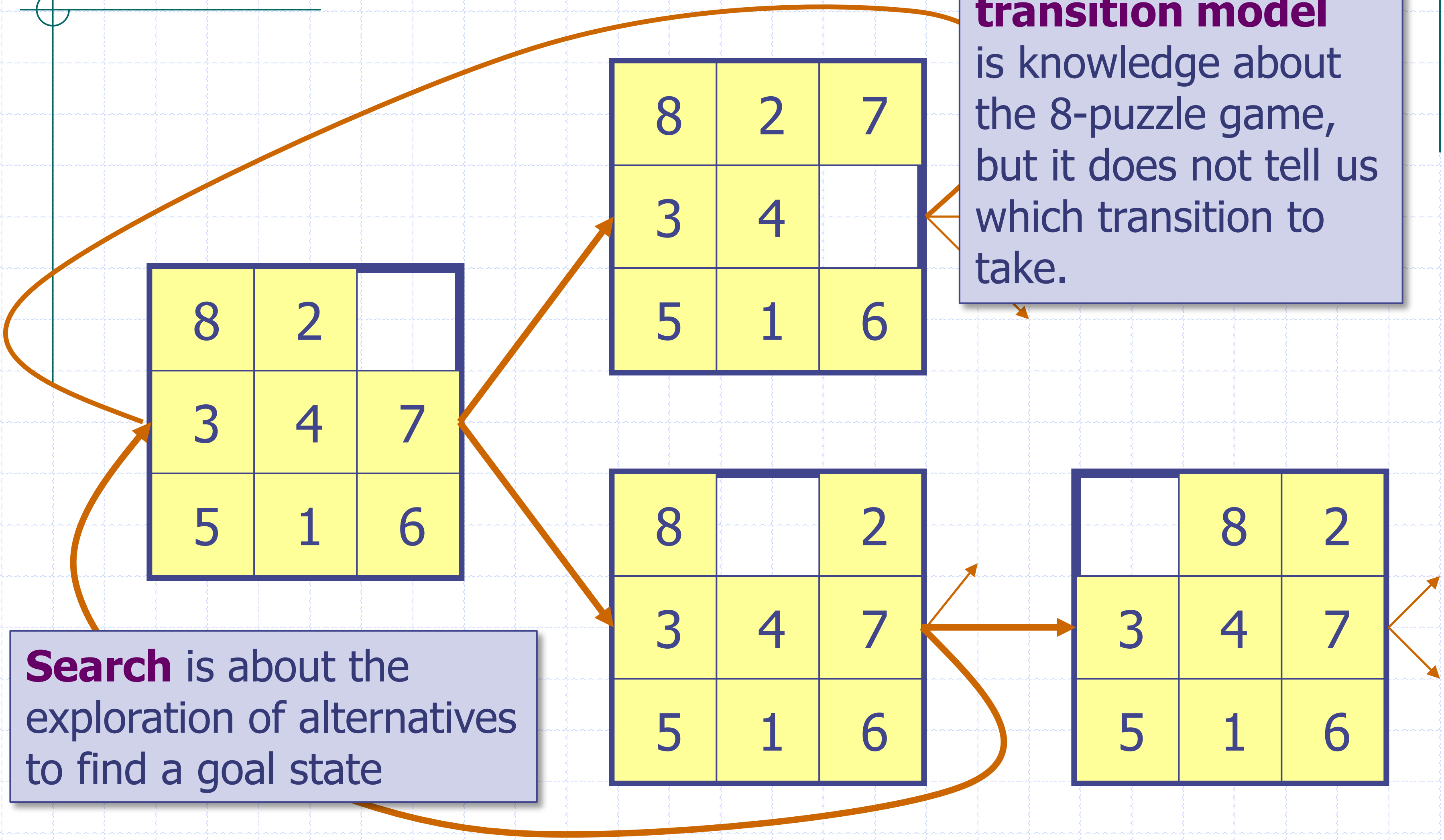
Initial state

1	2	3
4	5	6
7	8	

Goal state

State: Any arrangement of 8 numbered tiles and an empty tile on a 3x3 board

Example: 8-puzzle



Action set and transition model is knowledge about the 8-puzzle game, but it does not tell us which transition to take.

Search is about the exploration of alternatives to find a goal state

Example: 8-puzzle

Size of the state space = 9! = 362,880 (but half reachable)

15-puzzle ‣ ~2 x 10^{13}

24-puzzle ‣ ~1 x 10^{25}

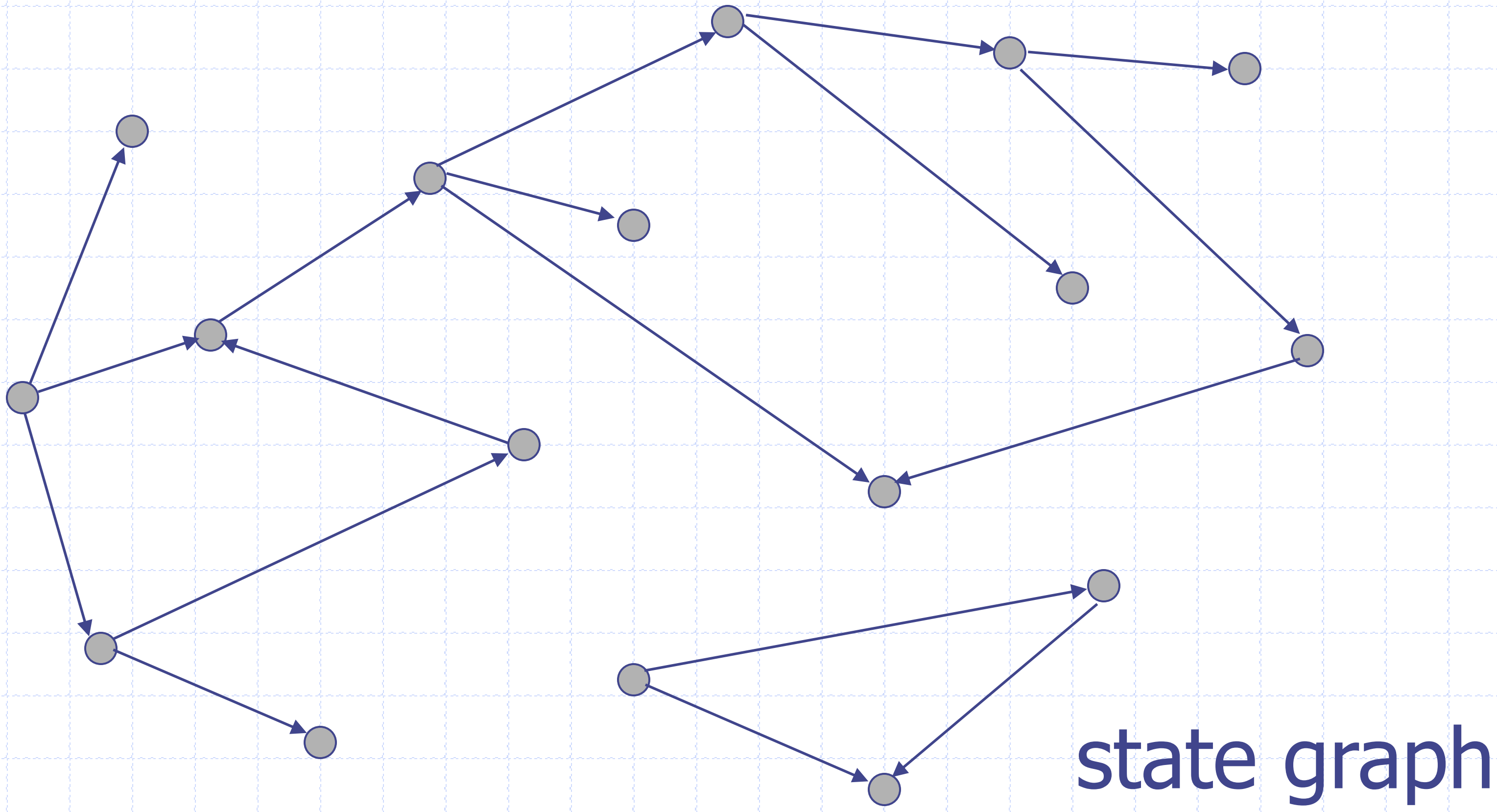
0.36 sec

24 days

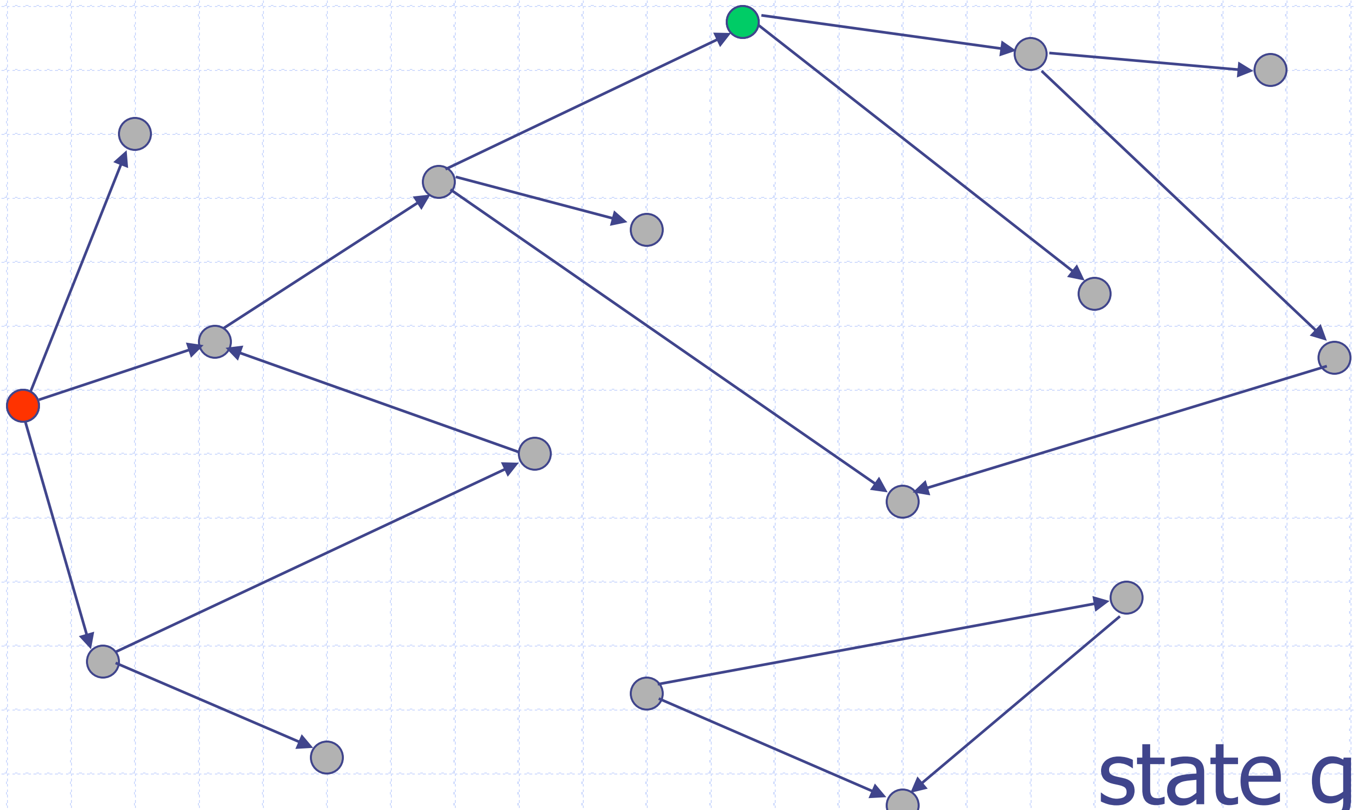
40 billion years

10 millions states/sec

Search of State Space

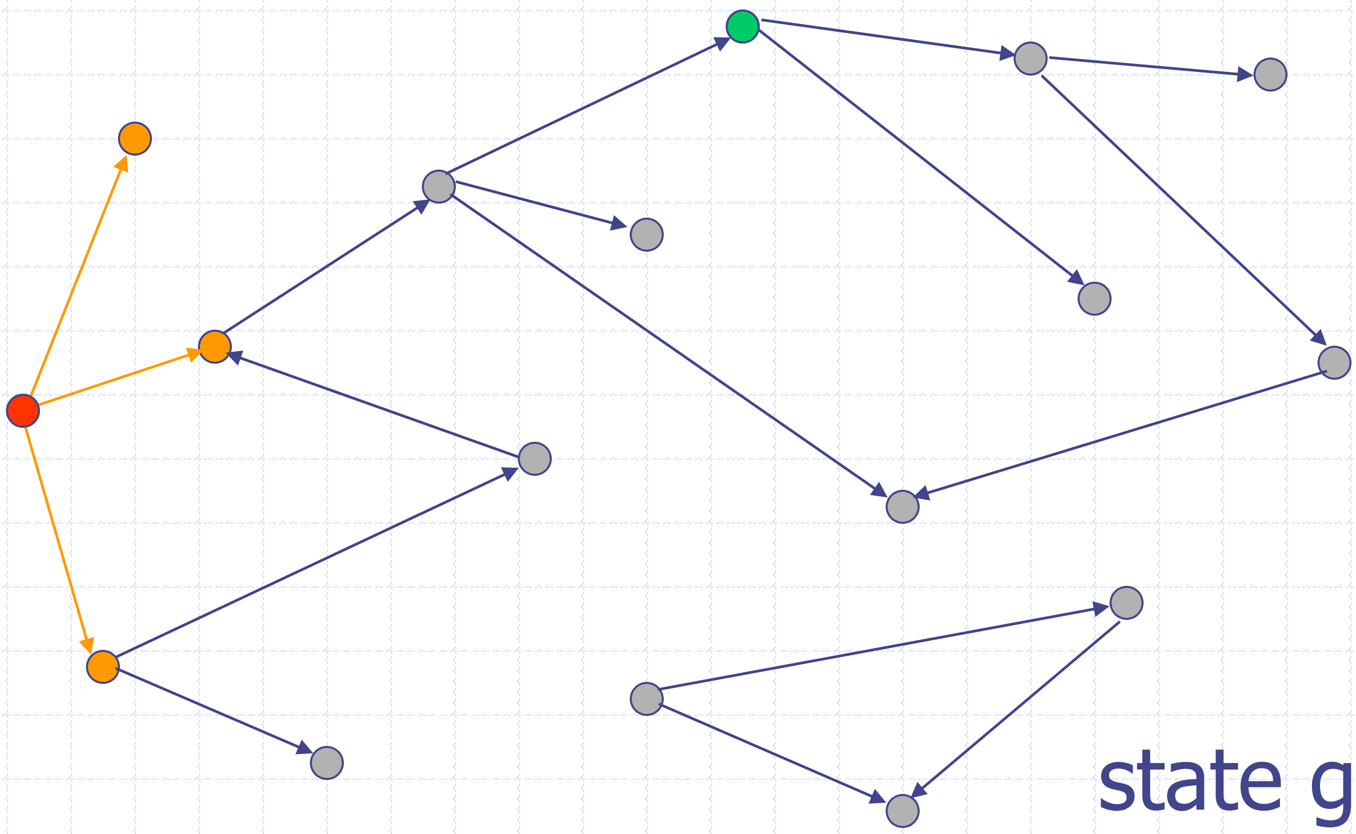


Search of State Space



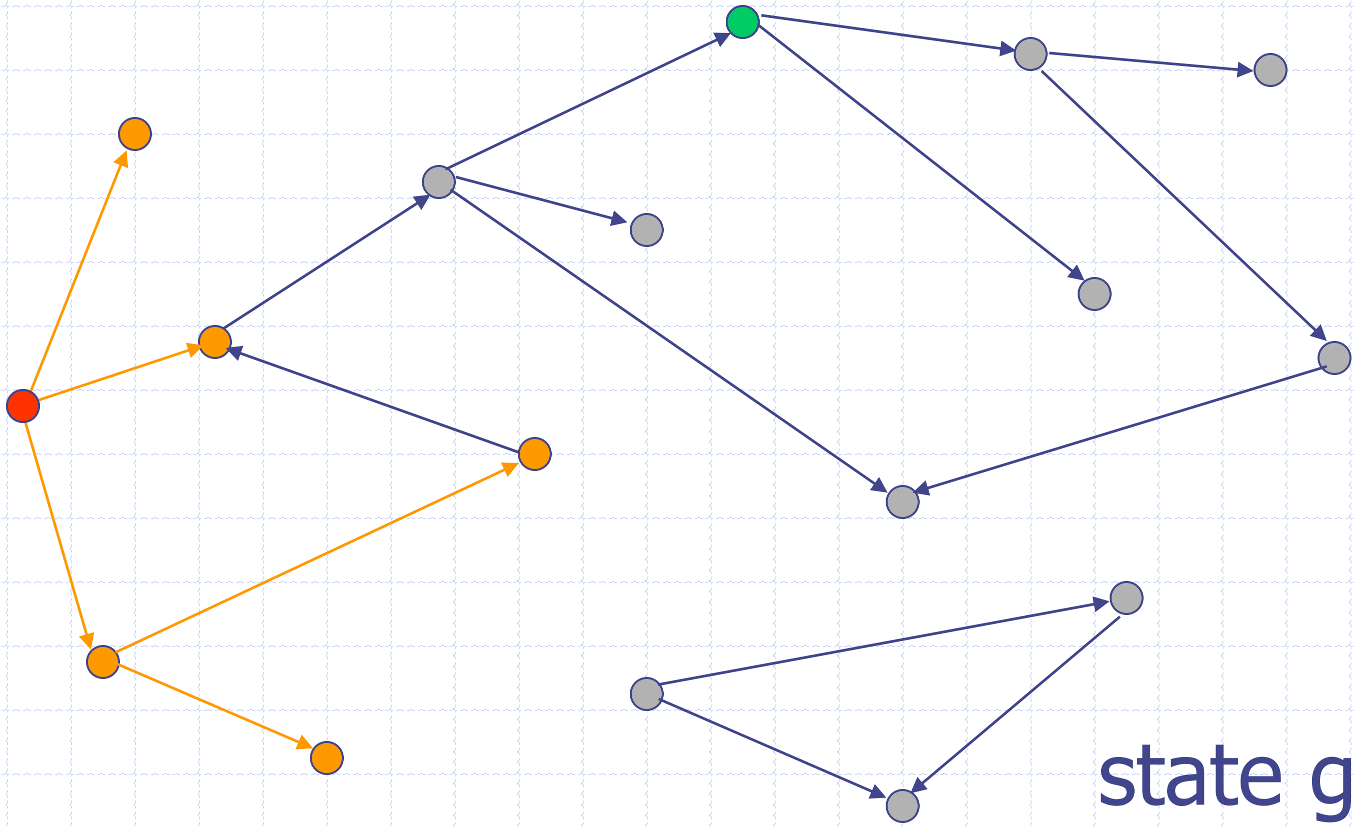
state graph

Search of State Space



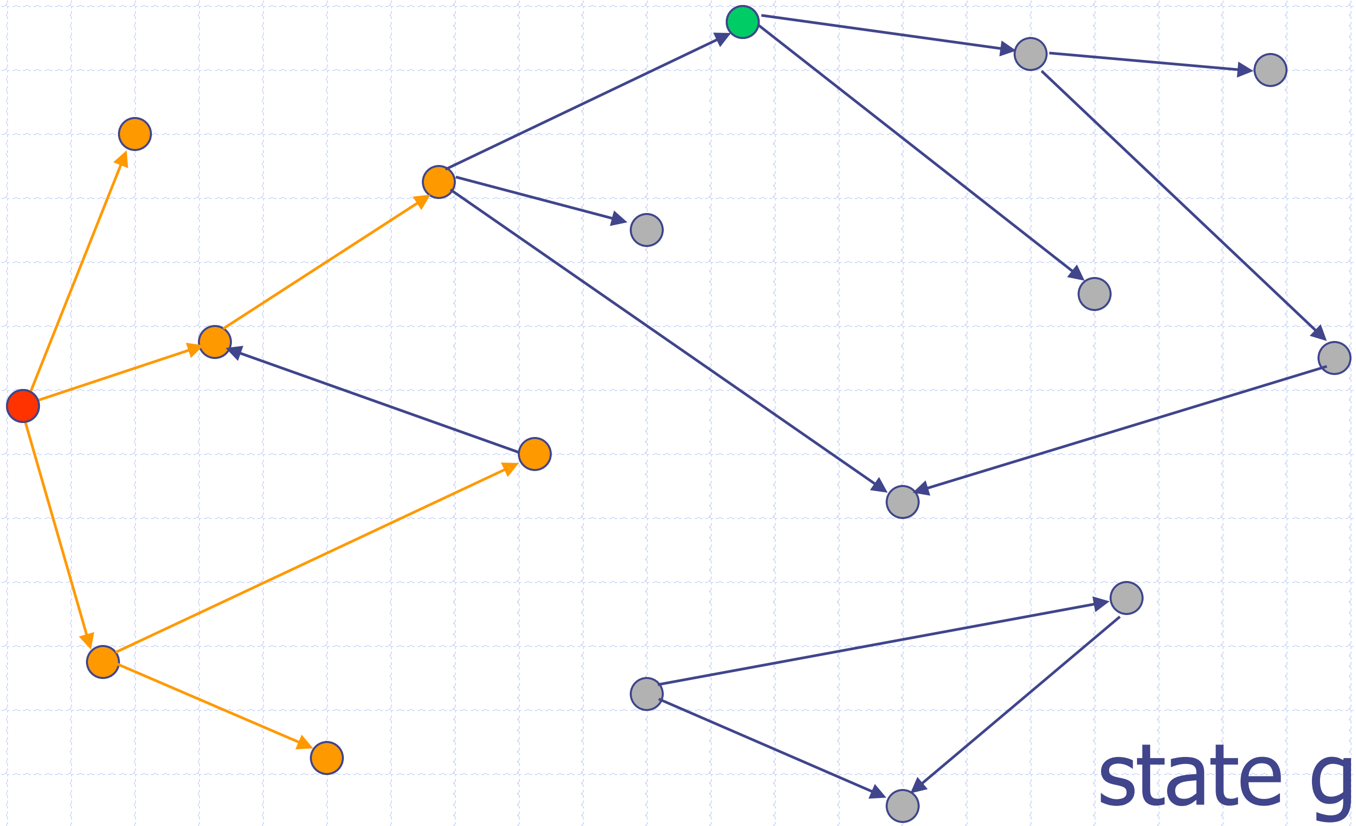
state graph

Search of State Space



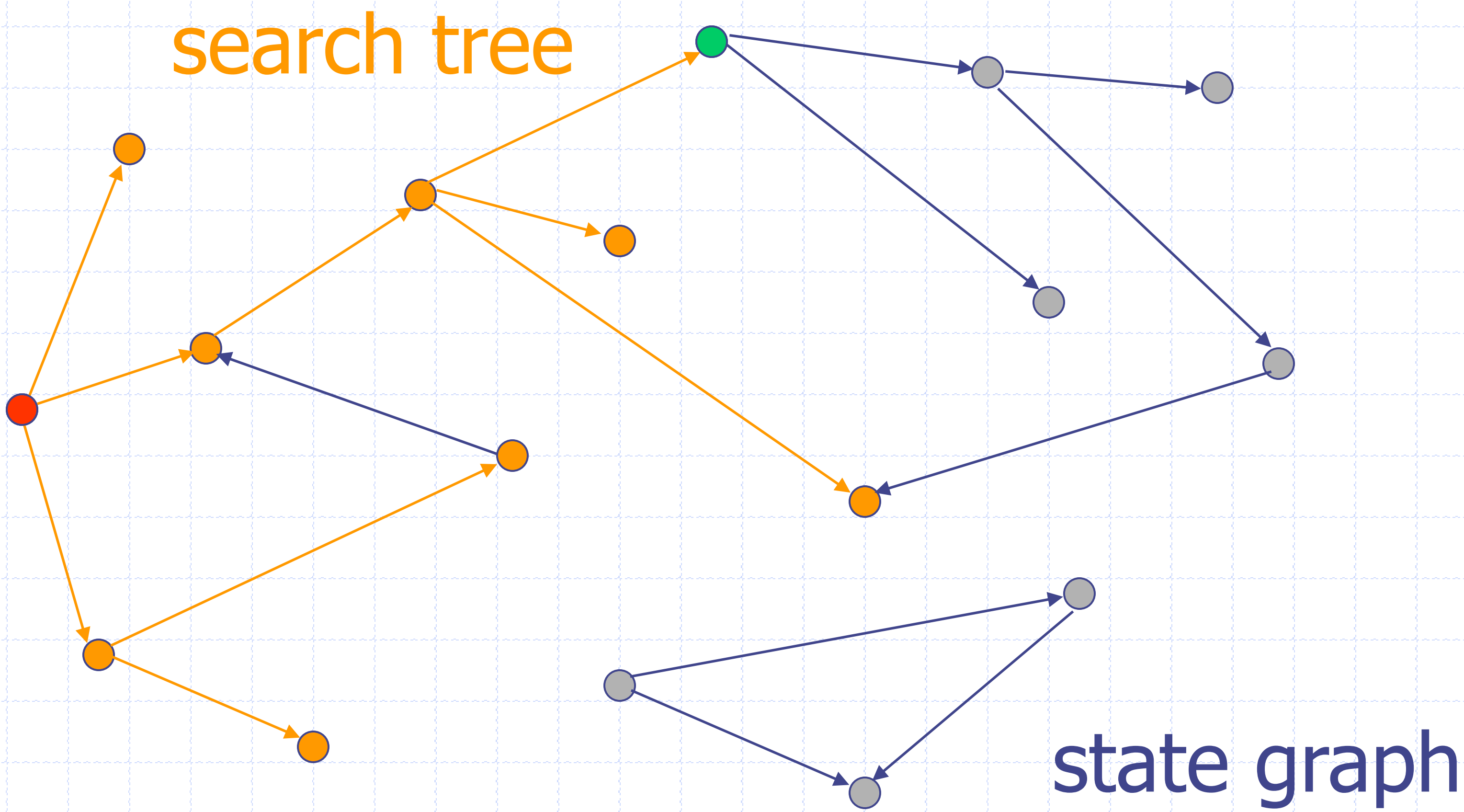
state graph

Search of State Space

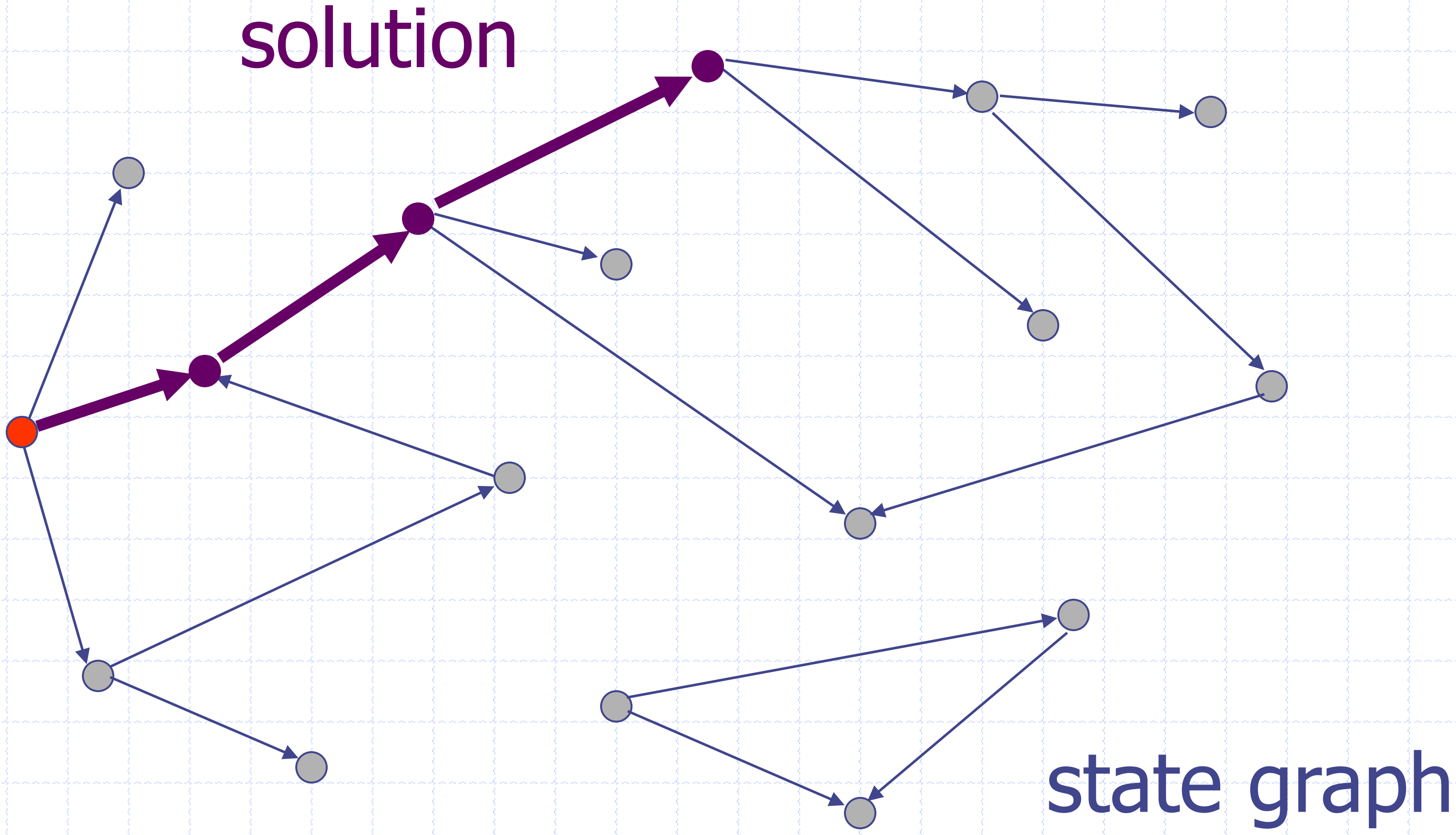


state graph

Search of State Space



Search of State Space



Search Problem

- Initial state
- Actions
- Transition model
- State space
- Goal test
- Path cost

Search Problem

- Initial state:
  - usually the current state
  - sometimes one or several hypothetical states ("what if …")
- Actions
- Transition model
- State space
- Goal test
- Path cost

Search Problem

- Initial state
- Actions:
  - possible actions available to agent
- Transition model
- State space
- Goal test
- Path cost

Search Problem

- Initial state
- Actions
- Transition model:
  - Result of doing action in state
  - Successor:
    reachable state by single action from current state
- State space
- Goal test
- Path cost

Search Problem

- Initial state
- Actions
- Transition model
- State space (implicitly defined by the above):
  - each state is an abstract representation of the environment
  - the state space is discrete
- Goal test
- Path cost

Search Problem

- Initial state
- Actions
- Transition model
- State space
- Goal test:
  - sometimes the description of a state
  - usually a condition
- Path cost

Search Problem

- Initial state
- Actions
- Transition model
- State space
- Goal test
- Path cost:
  - [path ▸ positive number]
  - usually, path cost = sum of step costs
  - e.g., number of moves of the empty tile

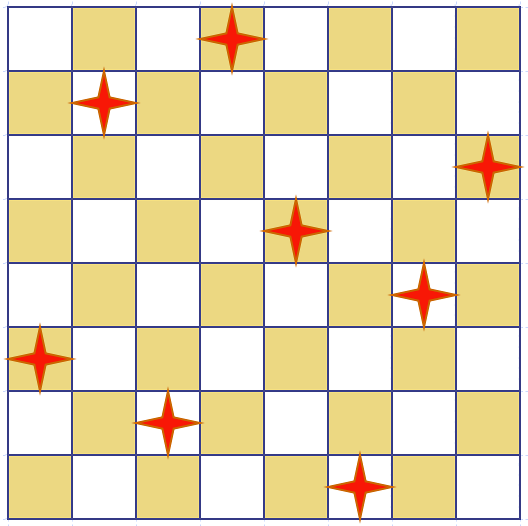
Simple Agent Algorithm

Problem-Solving-Agent

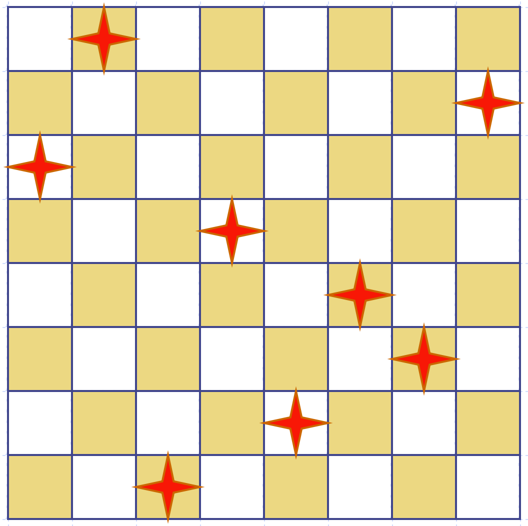
1. formulate: (abstraction!)
    1. initial-state ← sense/read state
    2. goal ← select/read goal
    3. actions ← select/read action models
    4. transition model ← select/read model
    5. problem ← (initial-state, goal, actions, transition model)
2. solution ← search(problem)
3. perform(solution)

Example: 8-queens

Place 8 queens in a chessboard so that no two queens are in the same row, column, or diagonal.

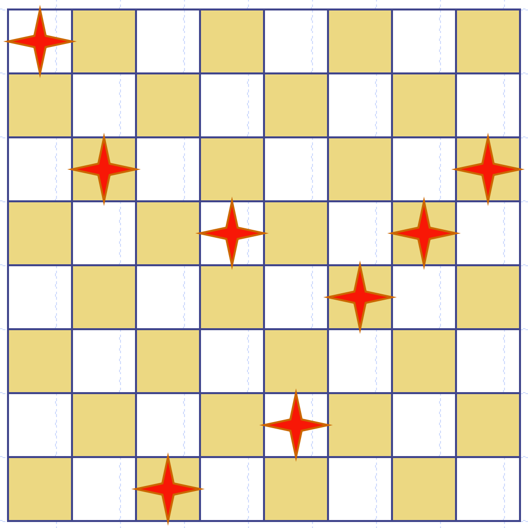


A solution



Not a solution

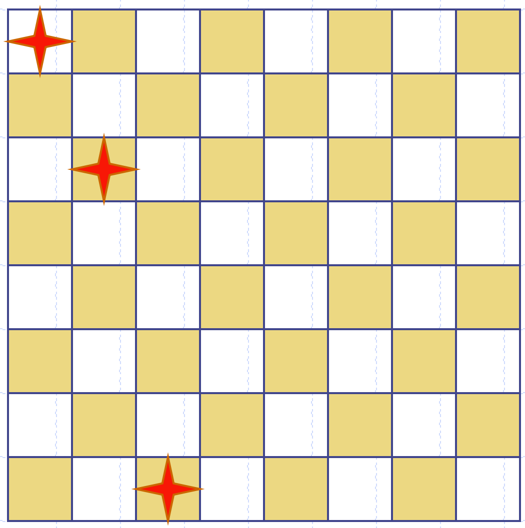
Example: 8-queens



Formulation #1:

- States: any arrangement of 0 to 8 queens on the board
- Initial state: 0 queens on the board
- Actions: add a queen in any empty square
- Transition model: board contains queen
- Goal test: 8 queens on the board, none attacked
  - 64^8 states with 8 queens

Example: 8-queens

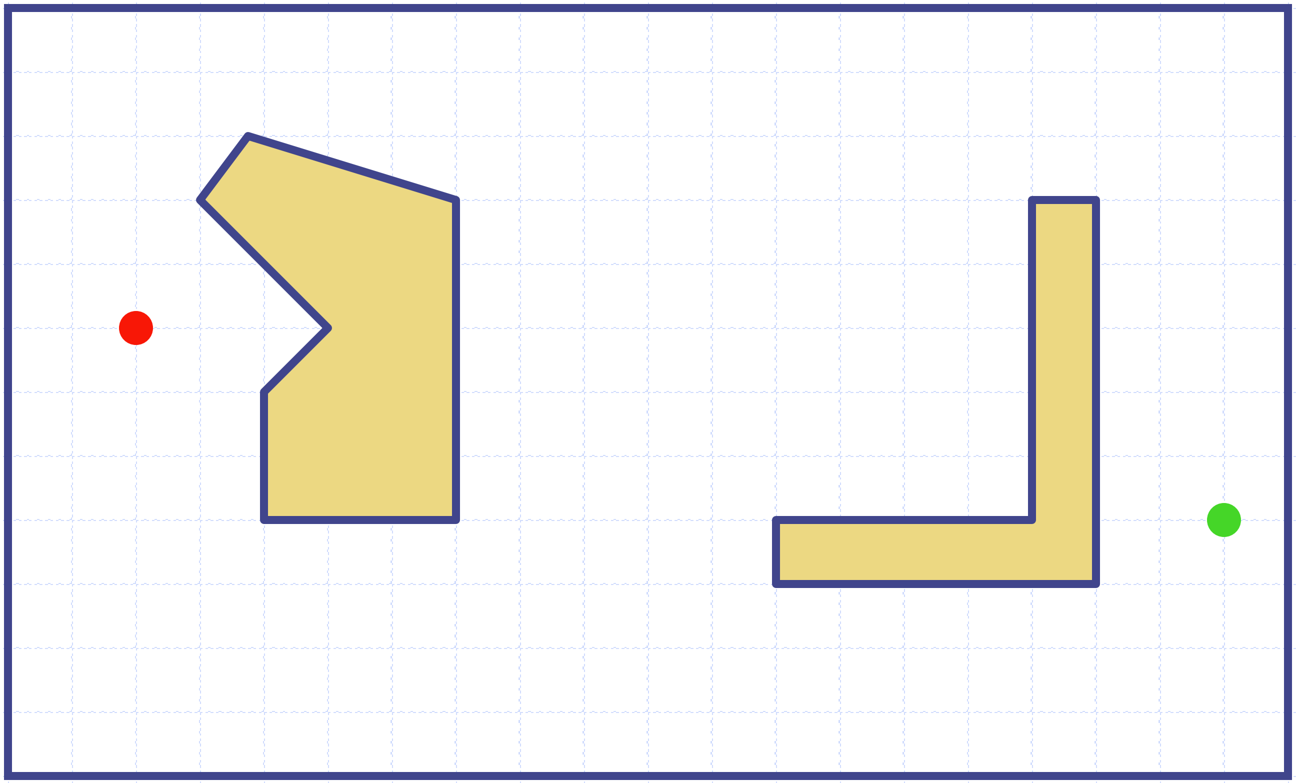


Formulation #2:

- States: any arrangement of $k = 0$ to 8 queens in the k leftmost columns with none attacked
- Initial state: 0 queens on the board
- Actions: add a queen to any square in the leftmost empty column such that it is not attacked by any other queen
- Transition model: board contains queen
- Goal test: 8 queens on the board

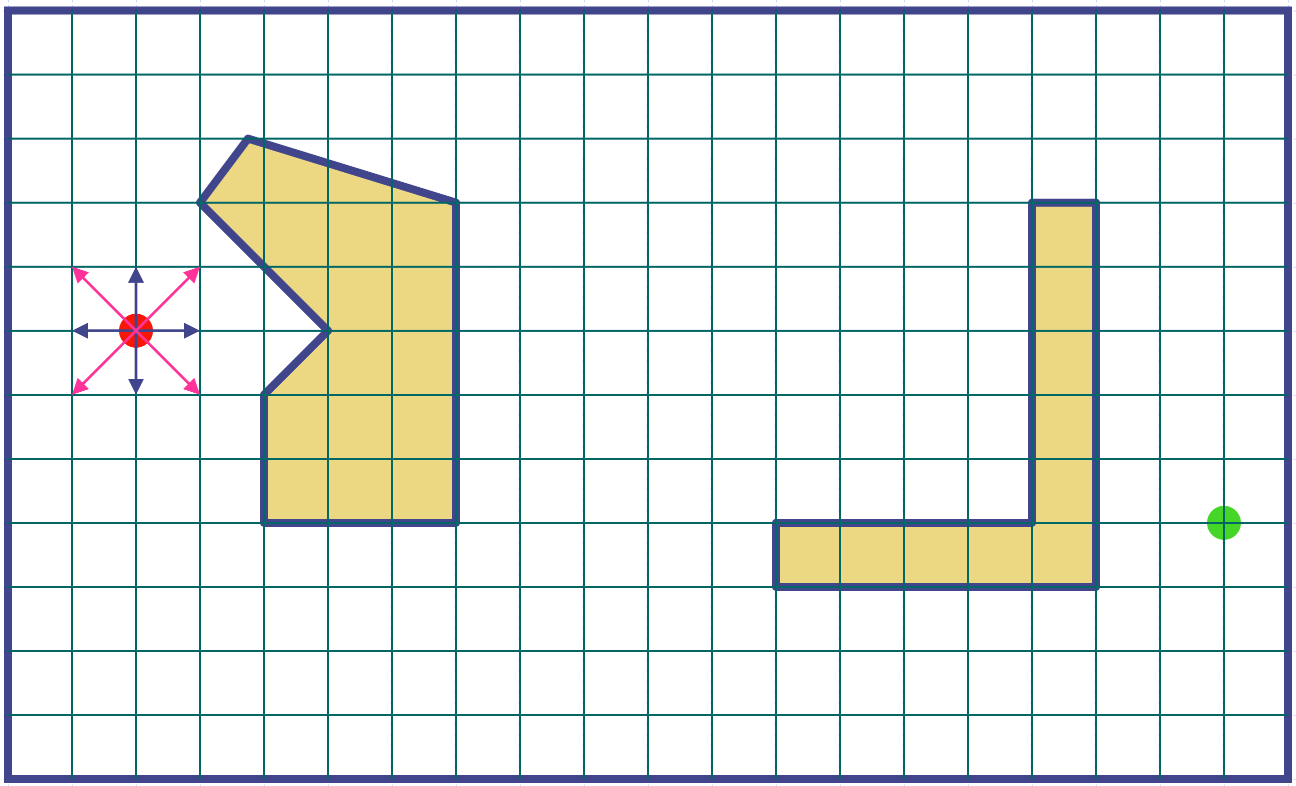
- 2,057 states

Example: Robot navigation



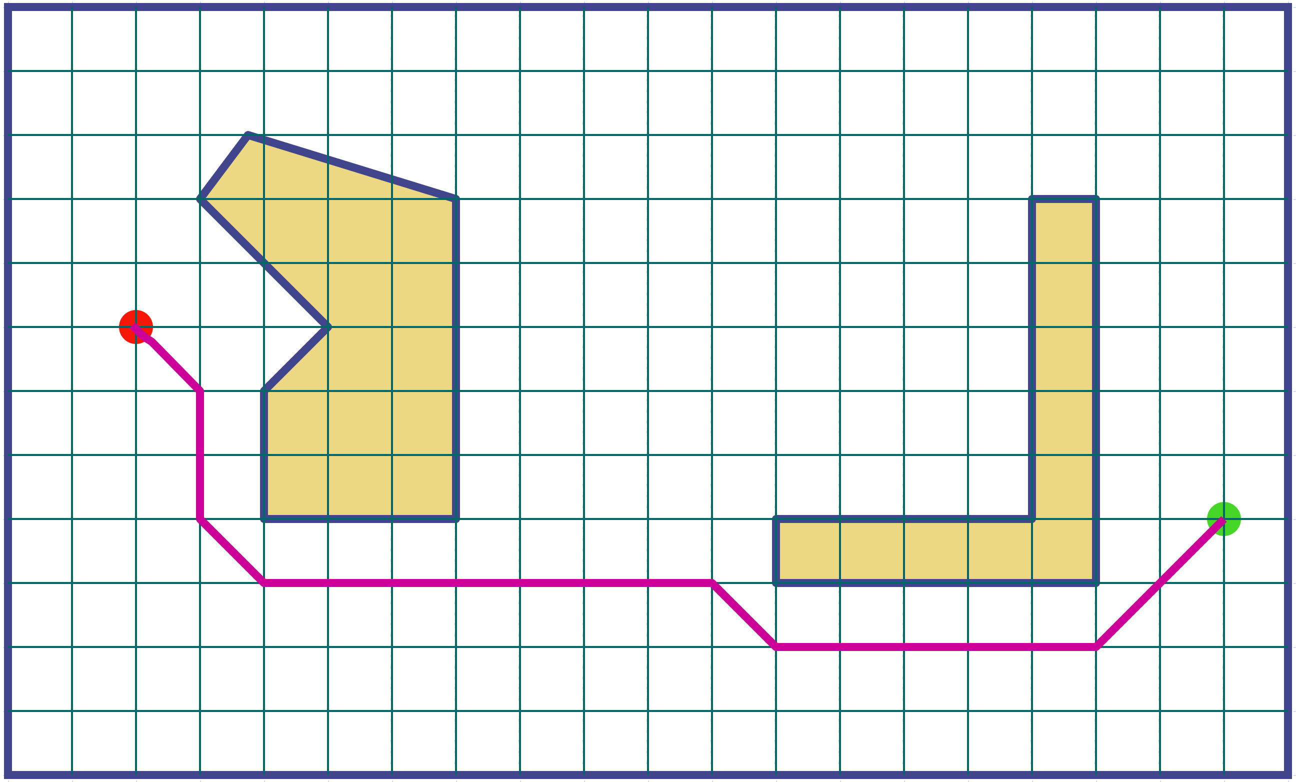
What is the state space?

Example: Robot navigation #1



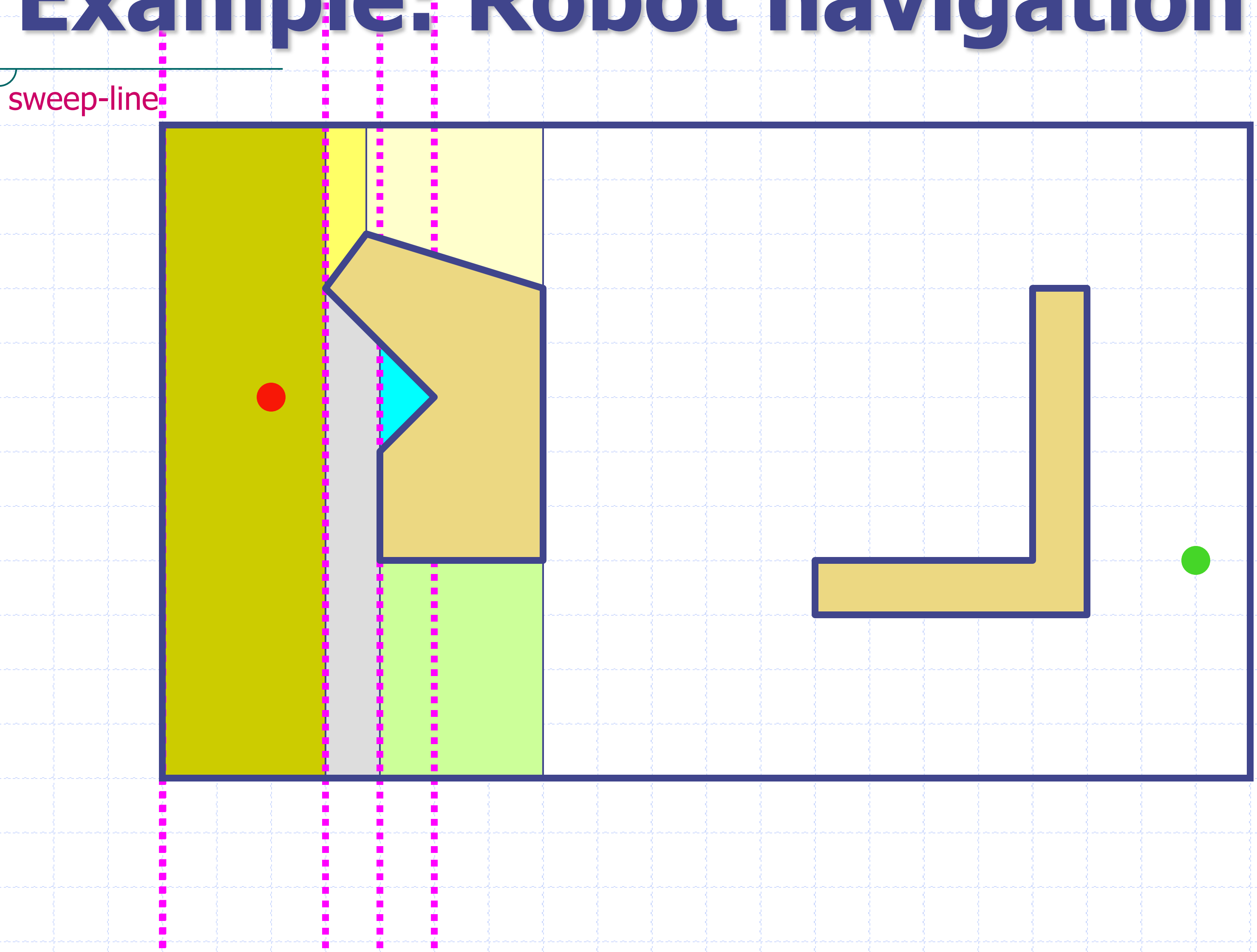
Cost of one horizontal/vertical step = 1
Cost of one diagonal step = $\sqrt{2}$

Example: Robot navigation #1

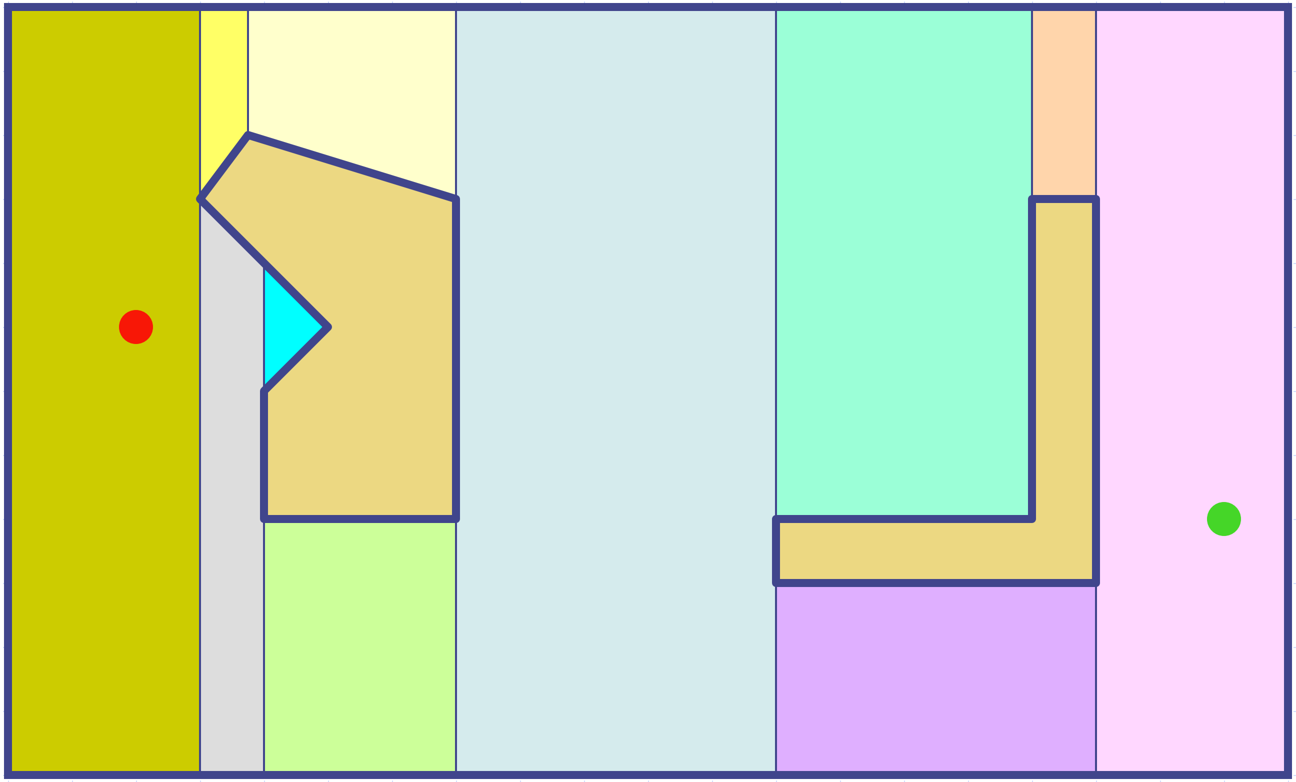


This path is the shortest in the discretized state space, but not in the original continuous space

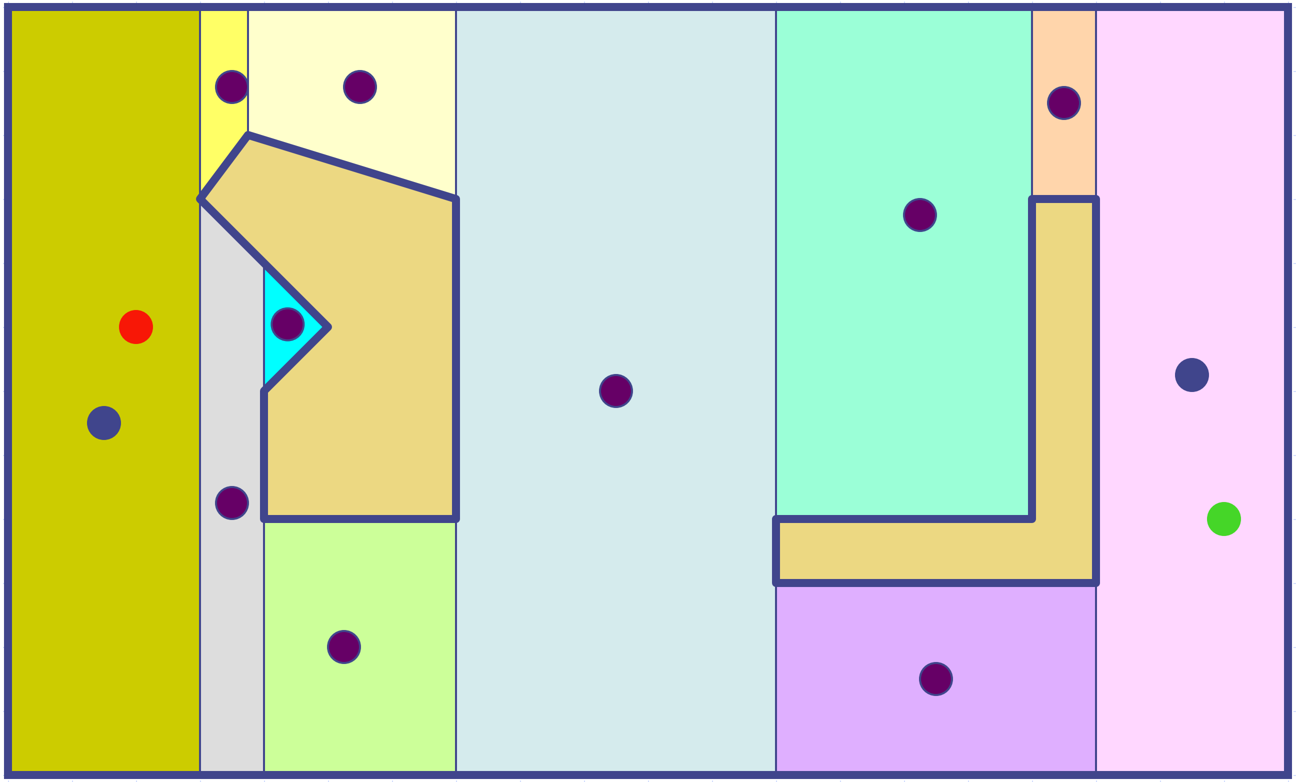
Example: Robot navigation #2



Example: Robot navigation #2

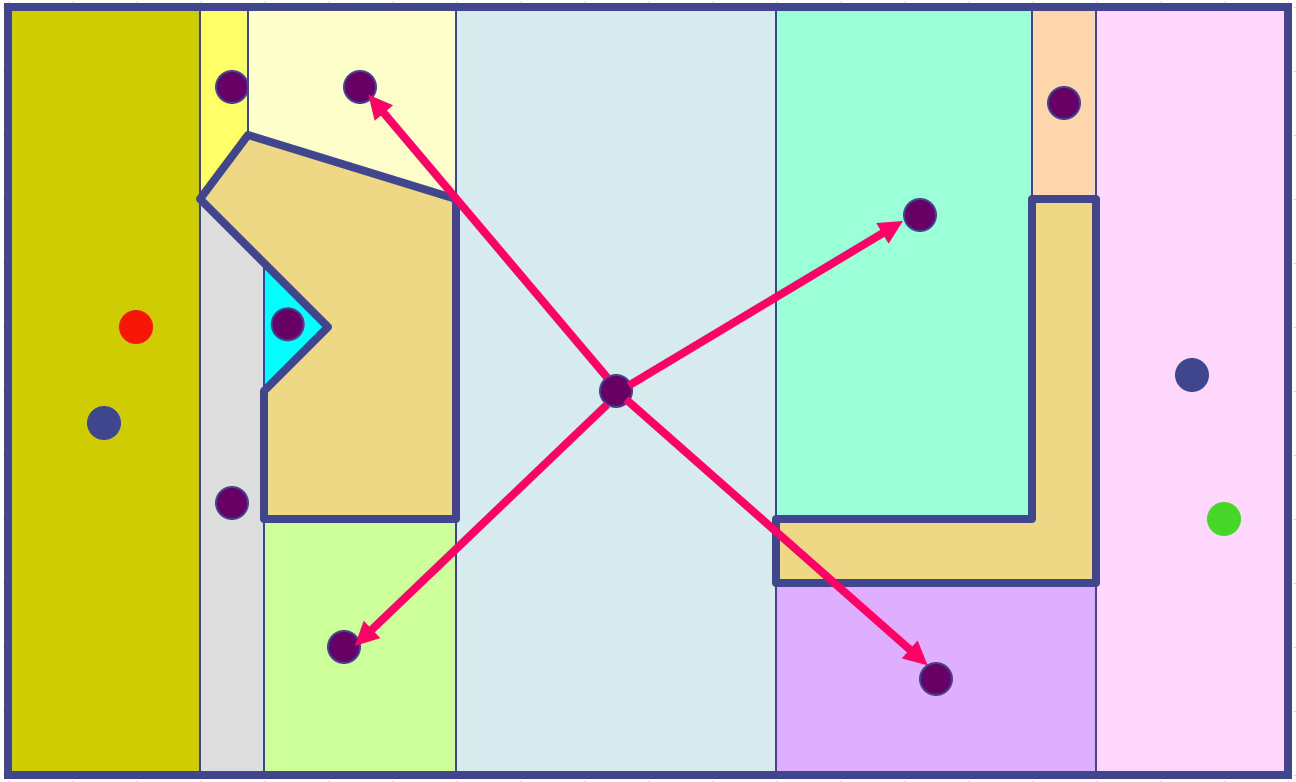


Example: Robot navigation #2



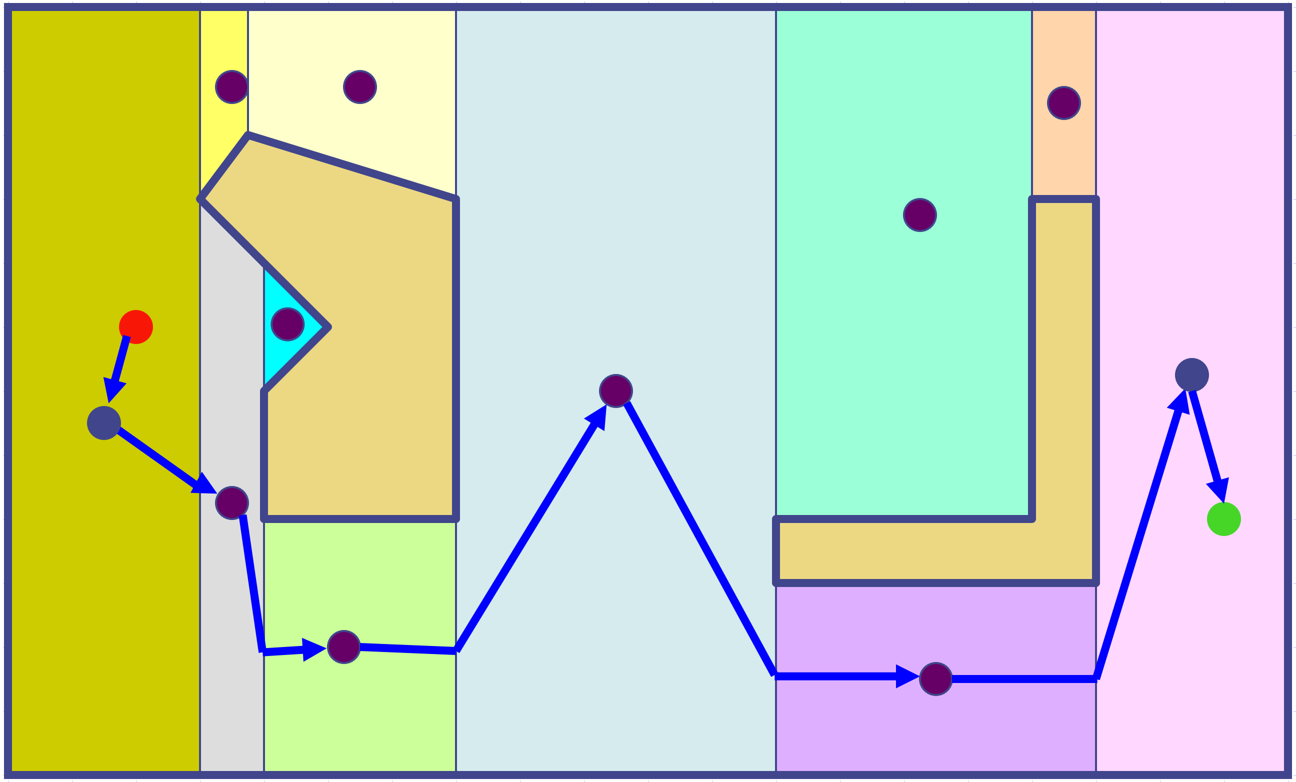
states

Example: Robot navigation #2



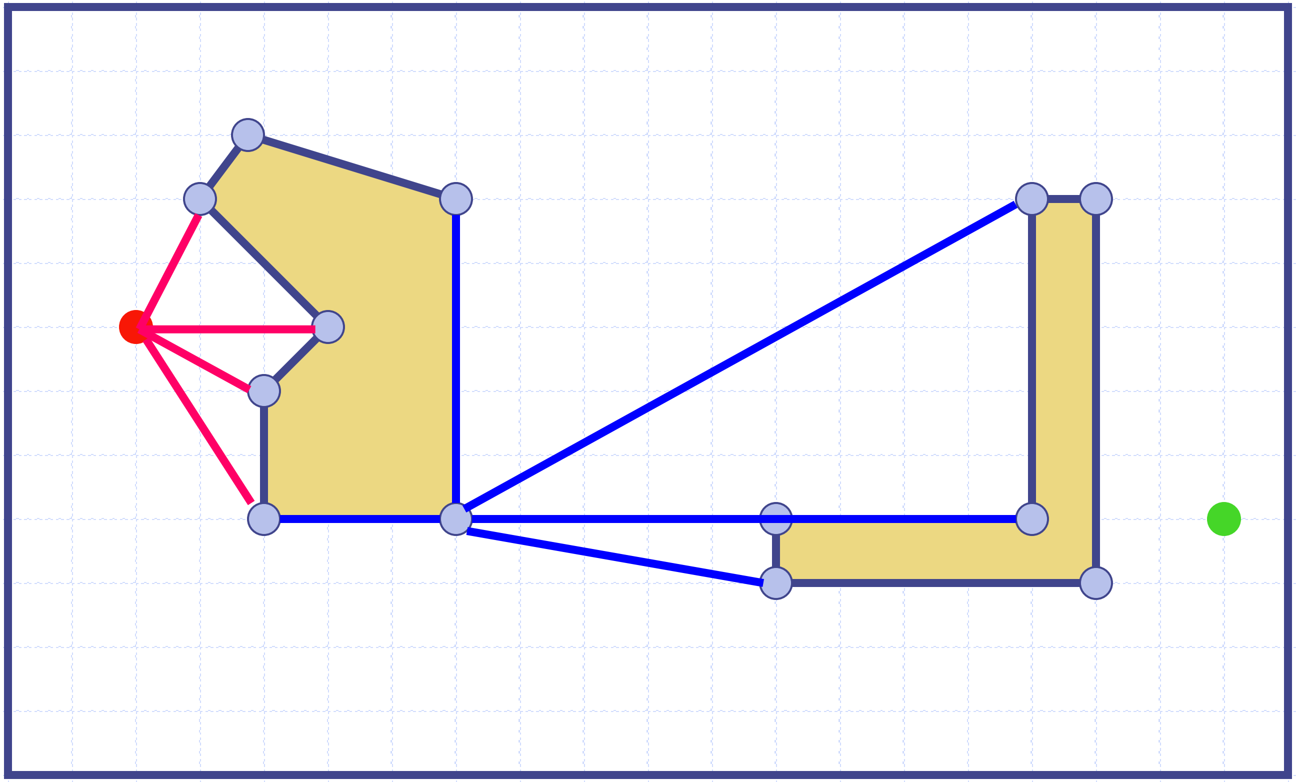
transition model

Example: Robot navigation #2



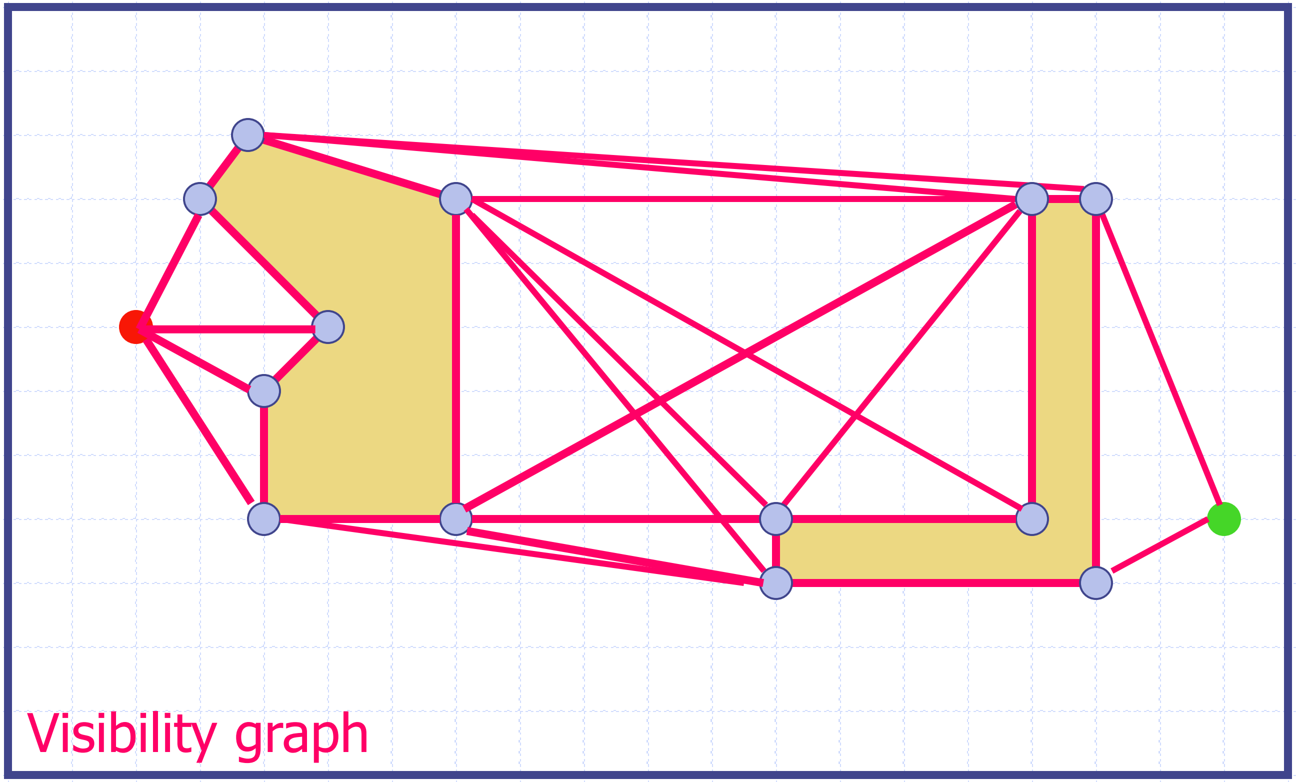
A path-smoothing post-processing step is usually needed to shorten the path further

Example: Robot navigation #3



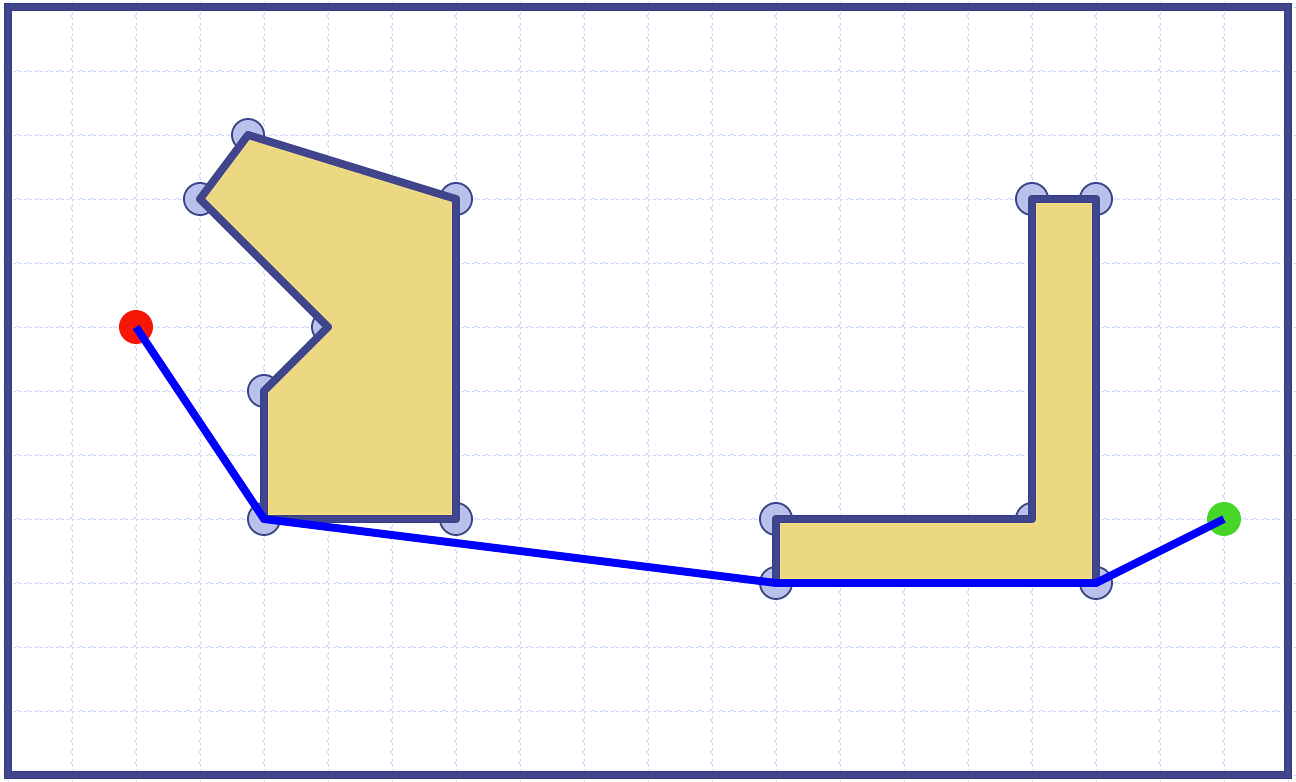
Cost of one step: length of segment

Example: Robot navigation #3



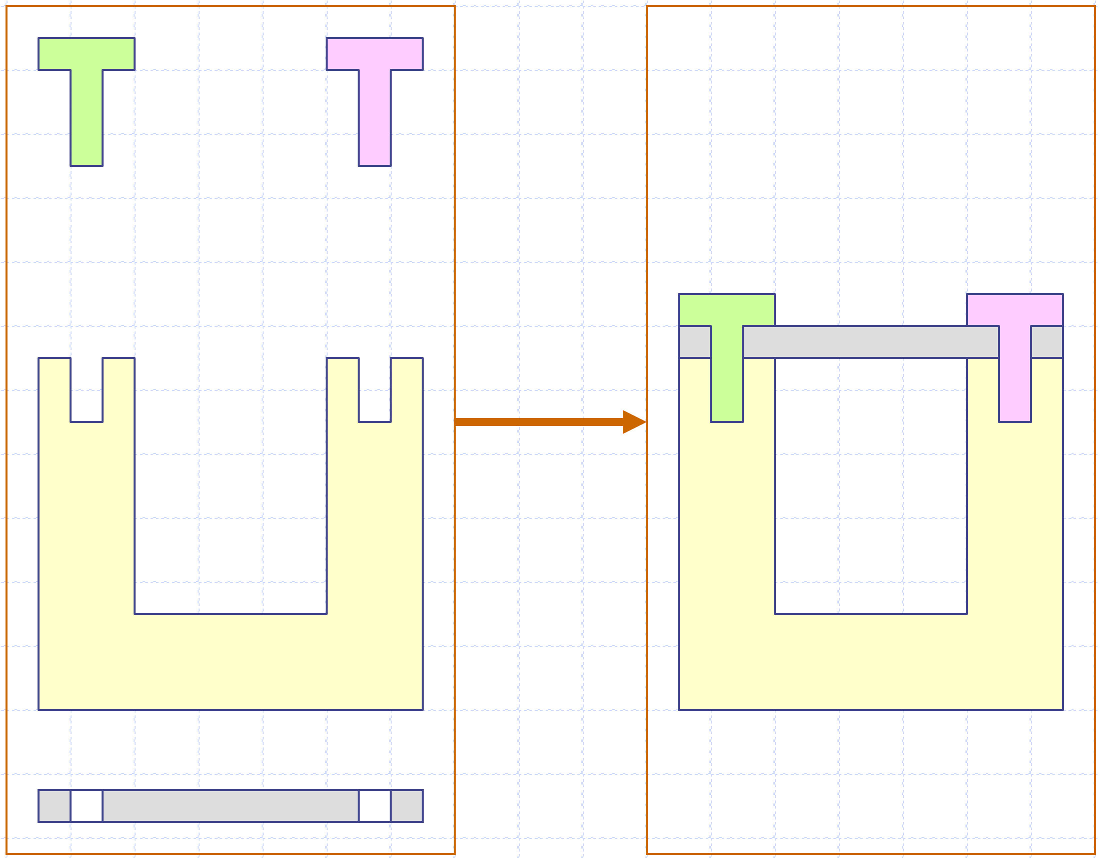
Cost of one step: length of segment

Example: Robot navigation #3

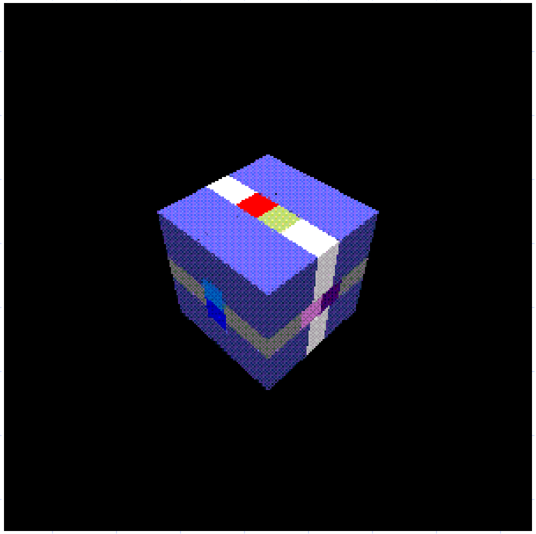


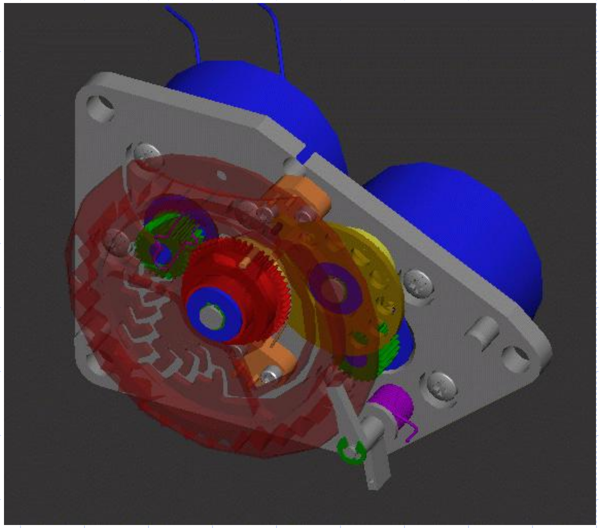
The shortest path in this state space is also the shortest in the original continuous space

Example: Assembly Planning



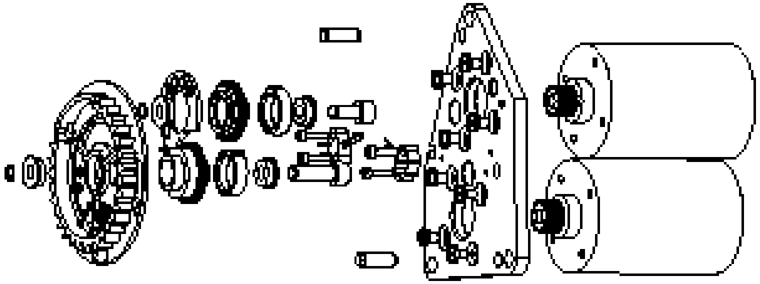
Example: Assembly Planning





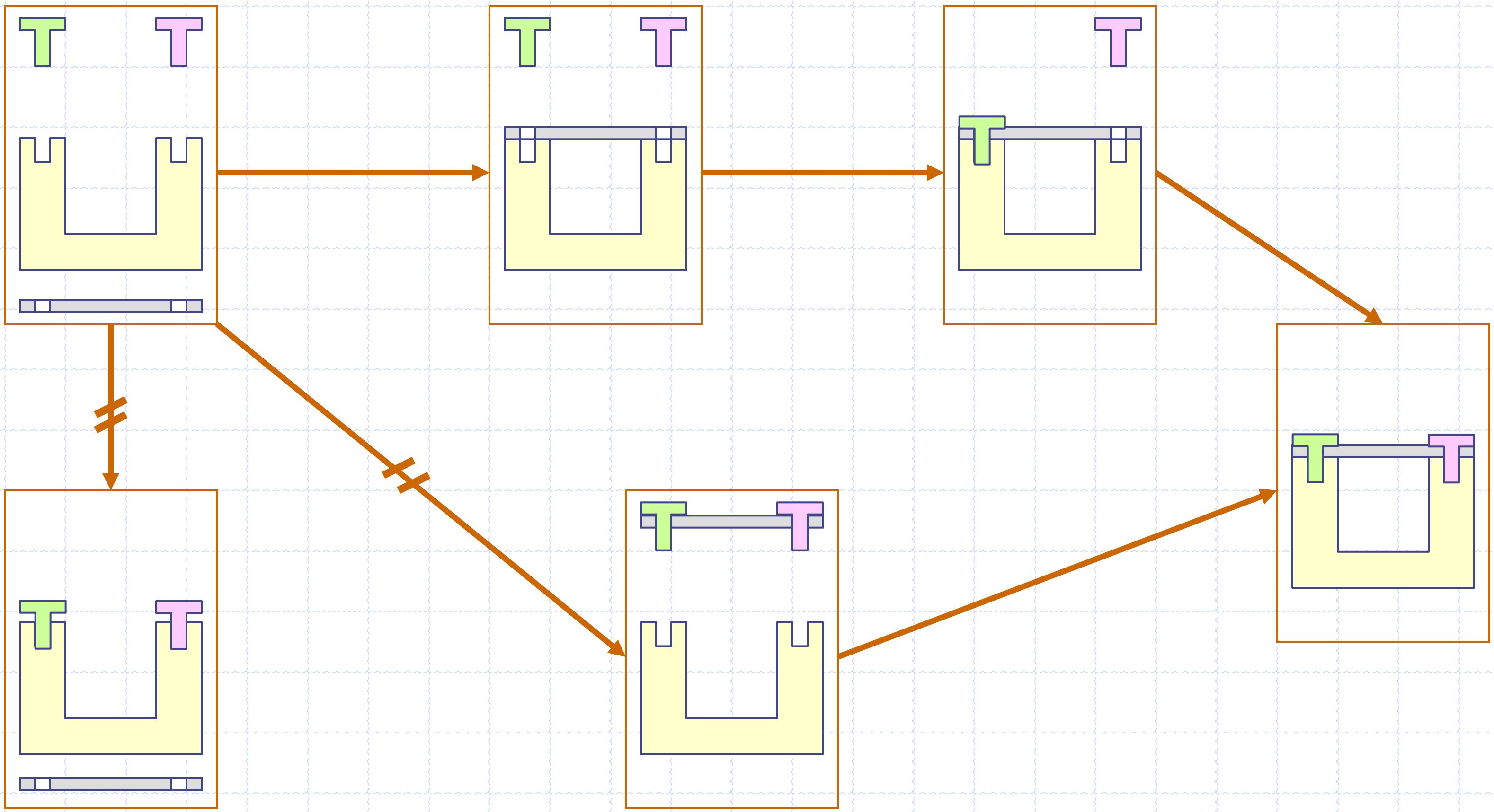
Example: Assembly Planning

- State: Collection of sub-assemblies
- Initial state: All sub-assemblies are individual parts



- Goal state: Complete assembly
- Actions: Merge two subassemblies (check for collision)
- Transition model: Merged assembly
- Cost function: Longest sequence of assembly operation

Example: Assembly Planning



Assumptions in Basic Search

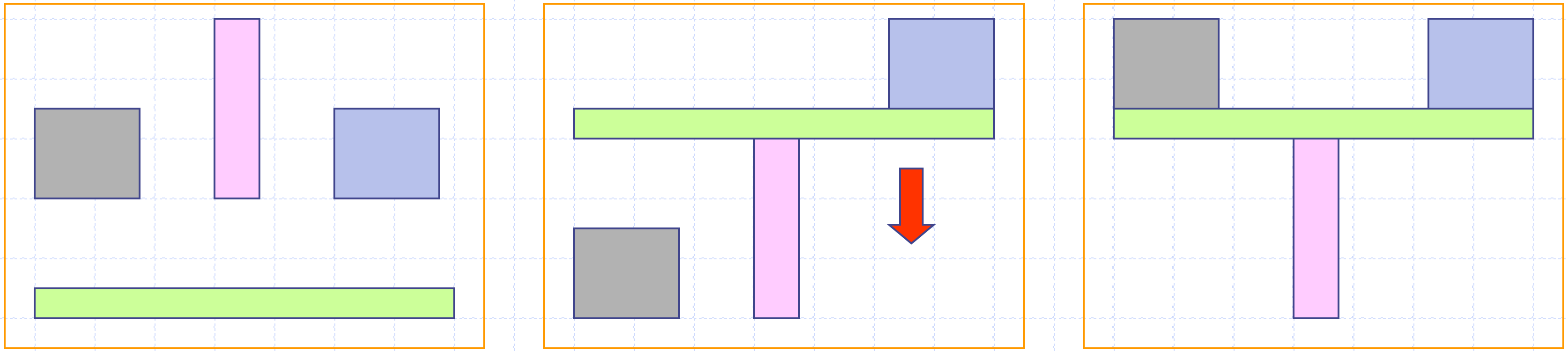
- The environment is static
- The environment is discretizable
- The environment is observable
- The actions are deterministic

Search Problem Formulation

- Real-world environment -> Abstraction

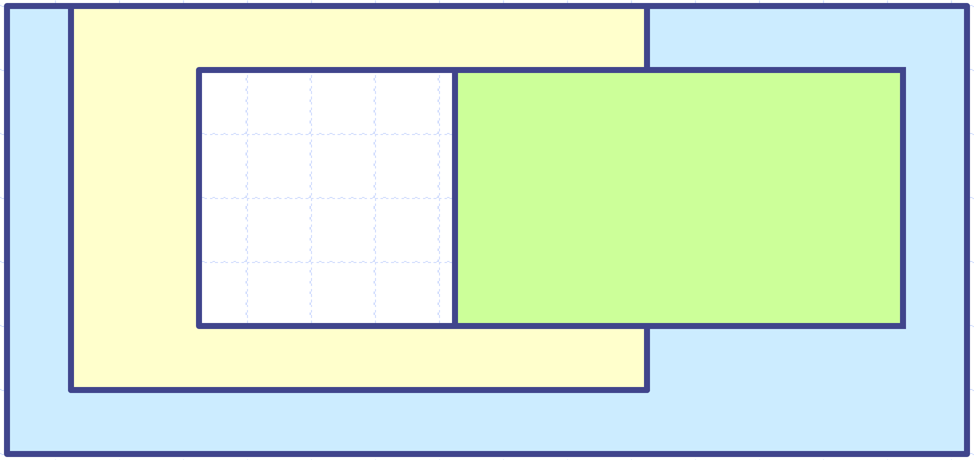
Search Problem Formulation

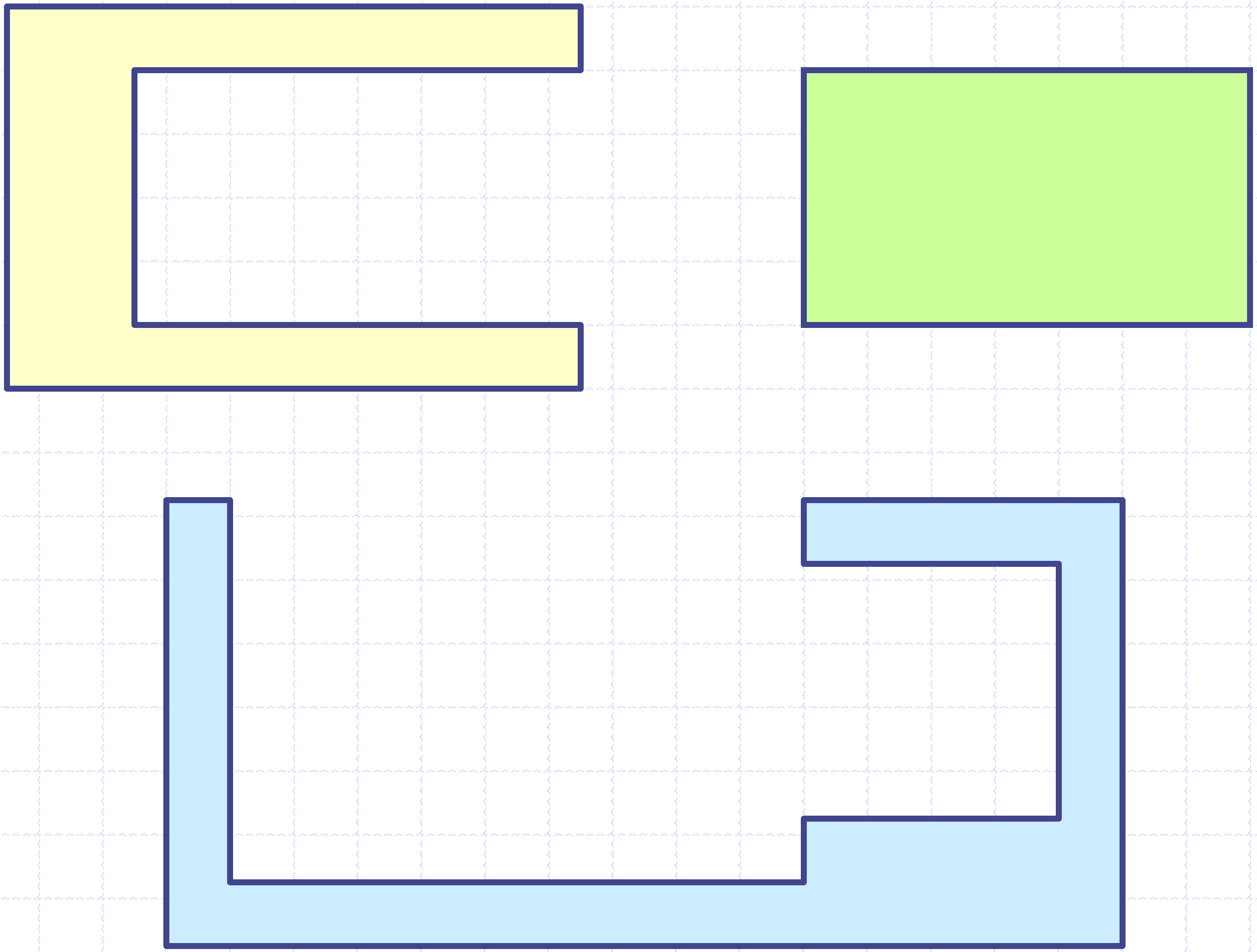
- Real-world environment -> Abstraction
  - Validity:
    - Can the solution be executed?

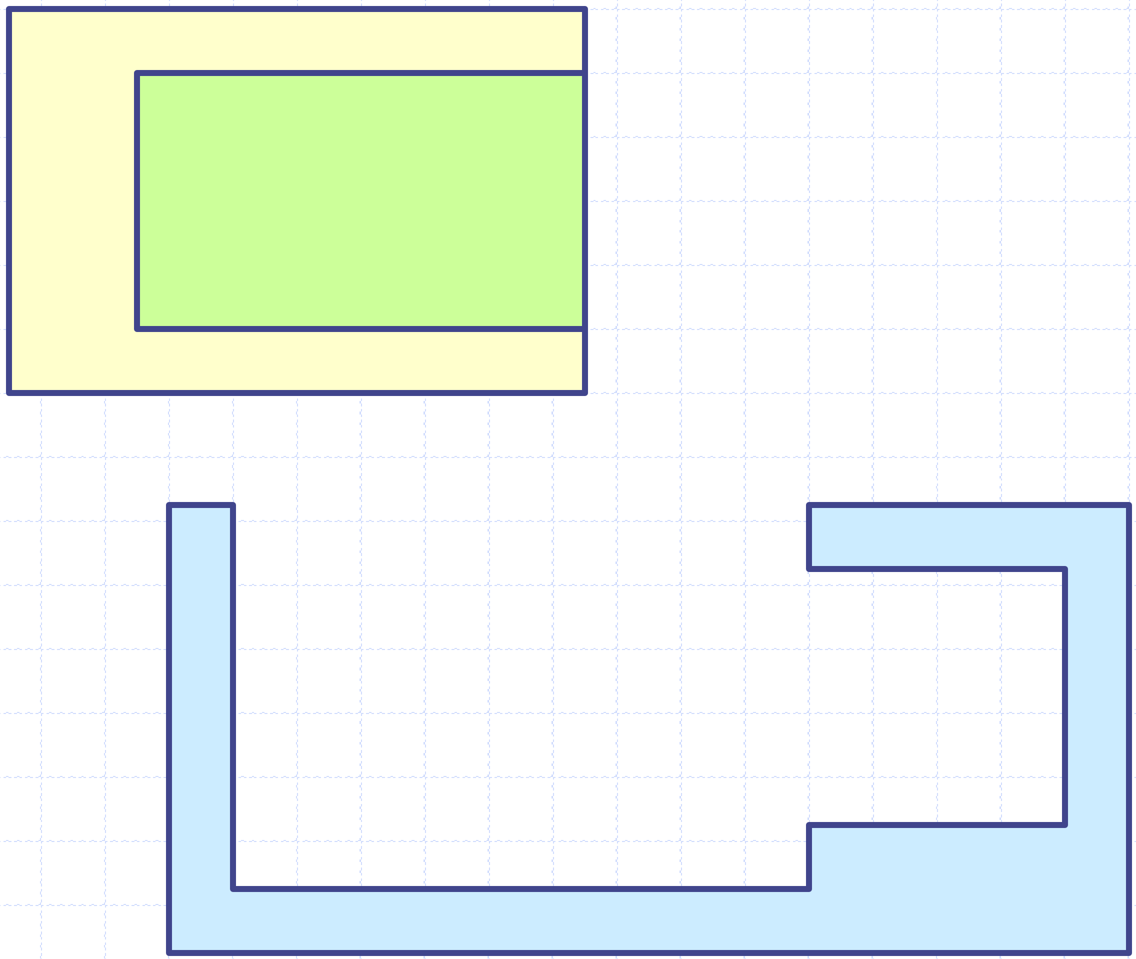


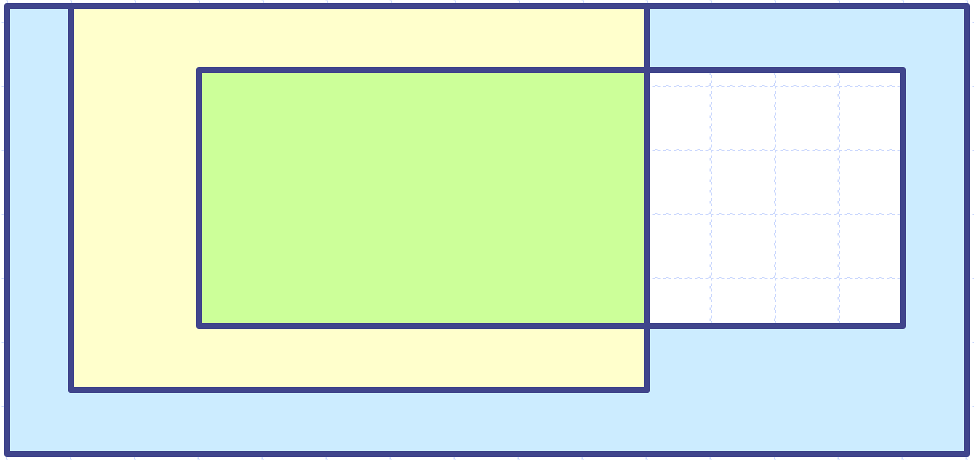
Search Problem Formulation

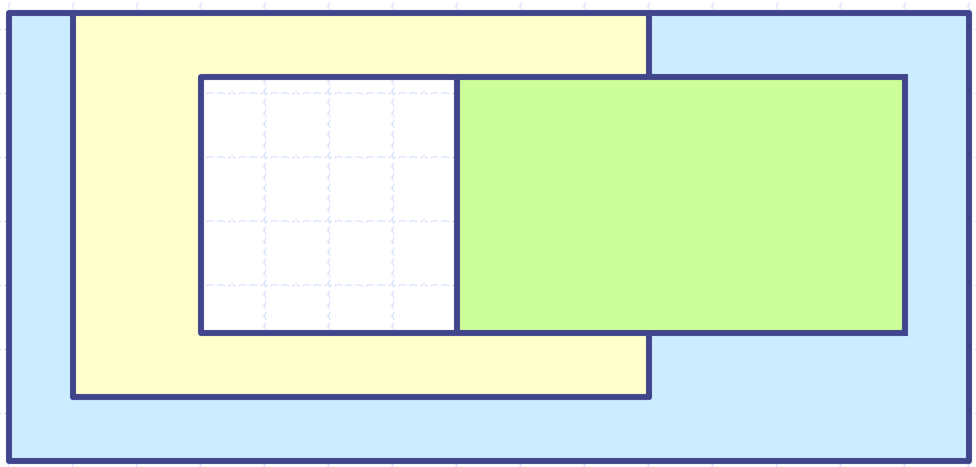
- Real-world environment -> Abstraction
  - Validity:
    - Can the solution be executed?
    - Does the state space contain the solution?











Search Problem Formulation

- Real-world environment -> Abstraction
  - Validity:
    - Can the solution be executed?
    - Does the state space contain the solution?
  - Usefulness
    - Is the abstract problem easier than the real-world problem?

Search Problem Formulation

- Real-world environment □ Abstraction
  - Validity:
    - Can the solution be executed?
    - Does the state space contain the solution?
  - Usefulness
    - Is the abstract problem easier than the real-world problem?
- Without abstraction an agent would be swamped by the complexity of the real world

Search Problem Variants

- One or several initial states
- One or several goal states
- The solution is the path or a goal node
  - In the 8-puzzle problem, it is the path to a goal node
  - In the 8-queen problem, it is a goal node

Search Problem Variants

- One or several initial states
- One or several goal states
- The solution is the path or a goal node
- Any, or the best, or all solutions

Important Parameters

- Number of states in state space

8-puzzle	▸ 362,880	8-queens	▸ 2,057
15-puzzle	▸ 2×10^{13}	100-queens	▸ 10^{52}
24-puzzle	▸ 1×10^{25}		

There exist techniques to solve N-queens problems efficiently!

Stating a problem as a search problem is not always a good idea!

Important Parameters

- Number of states in state space
- Distribution of goal states
- Size of memory needed to store a state

Important Parameters

- Number of states in state space
- Distribution of goal states
- Size of memory needed to store a state
- Running time of the successor function

Applications

- Route finding: airline travel, networks
- Pipe routing, VLSI routing
- Pharmaceutical drug design
- Robot motion planning
- Video games

Summary

- Problem-solving agent
- State space, actions, transition model
- Search!
- Examples:
  - 8-puzzle, 8-queens, route finding, robot navigation, assembly planning
- Assumptions of basic search
- Important parameters