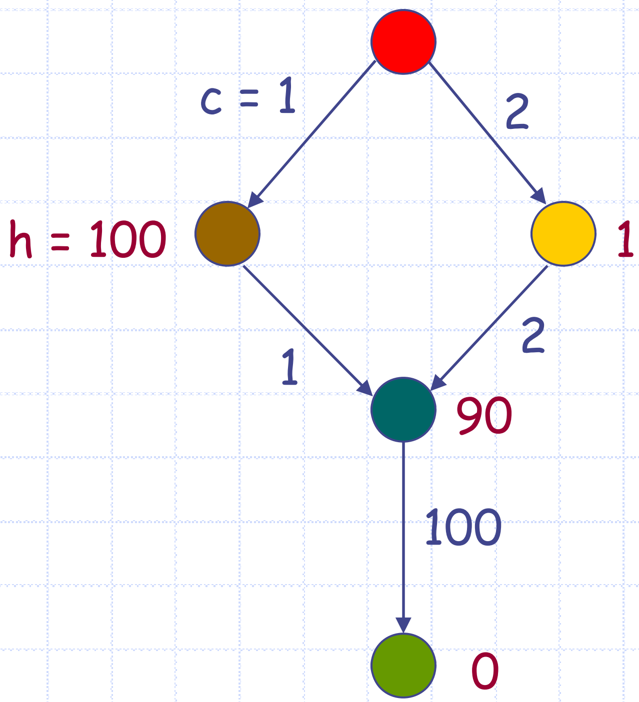
Heuristic (Informed) **Search B**

(Where we try to choose smartly)

Russell and Norvig:
Chap. 3, Sect. 3.5 - 3.6

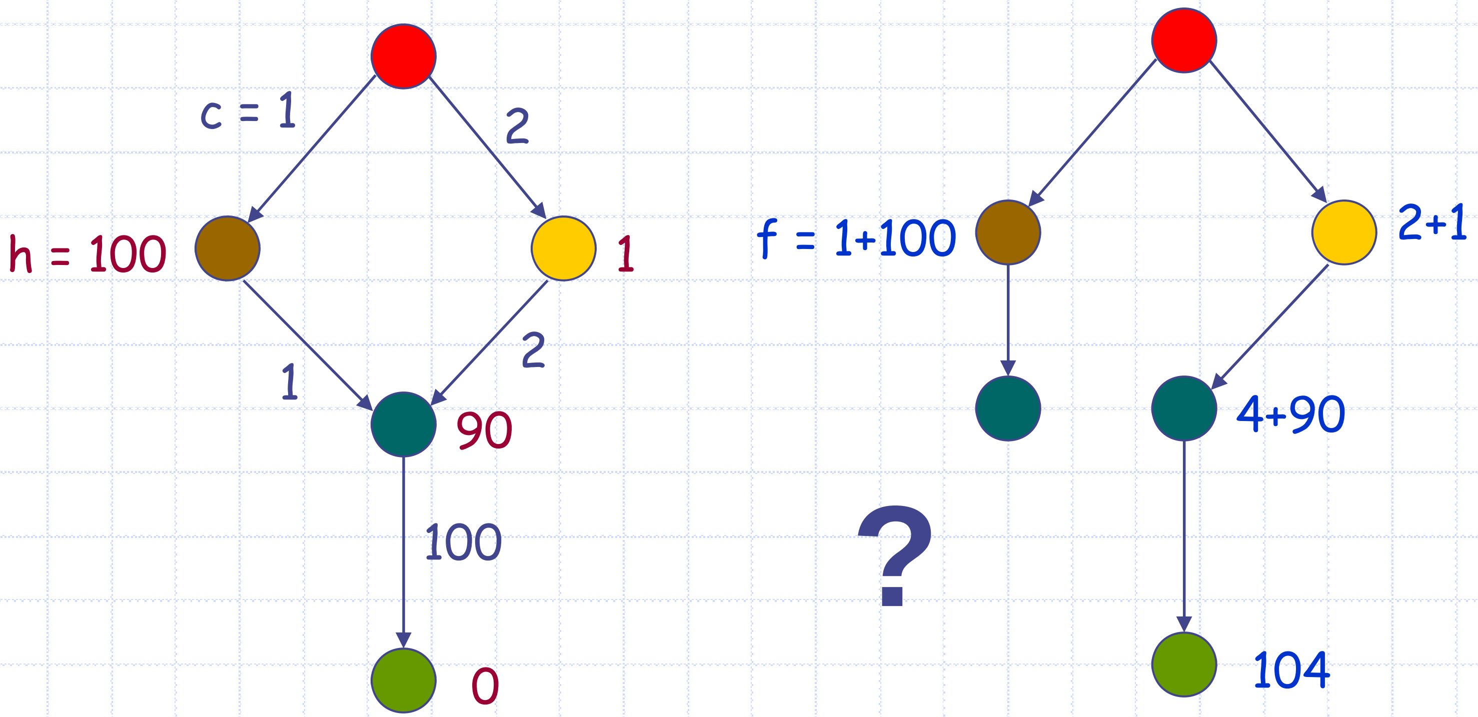
Slides adapted from Jean-Claude Latombe at Stanford University (used with permission)

Handling Revisited States



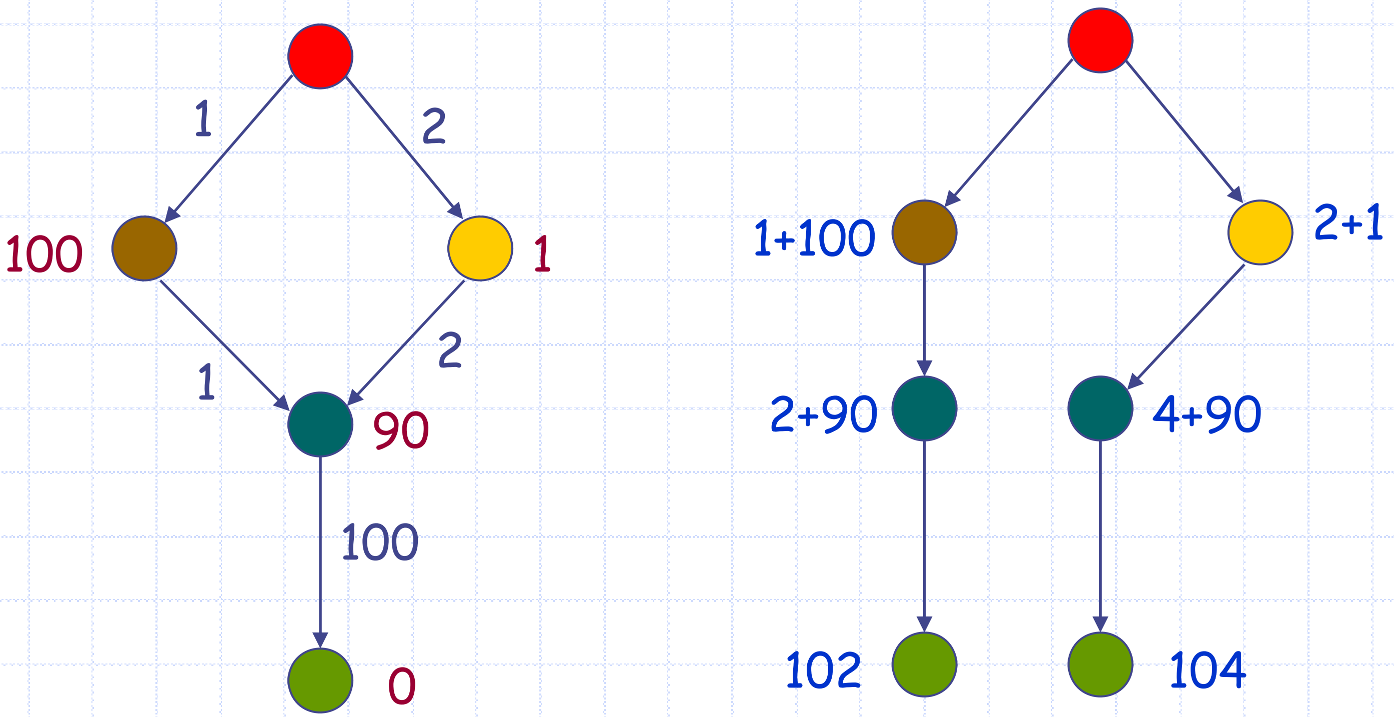
The heuristic **h** is clearly admissible

Handling Revisited States



If we discard this new node, then the search algorithm expands the goal node next and returns a non-optimal solution

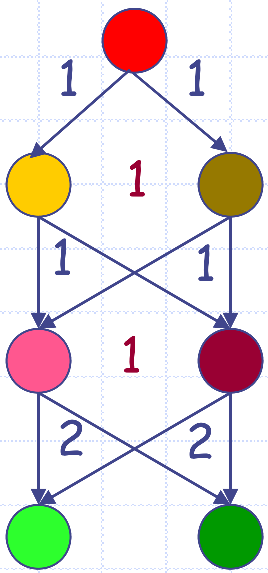
Handling Revisited States



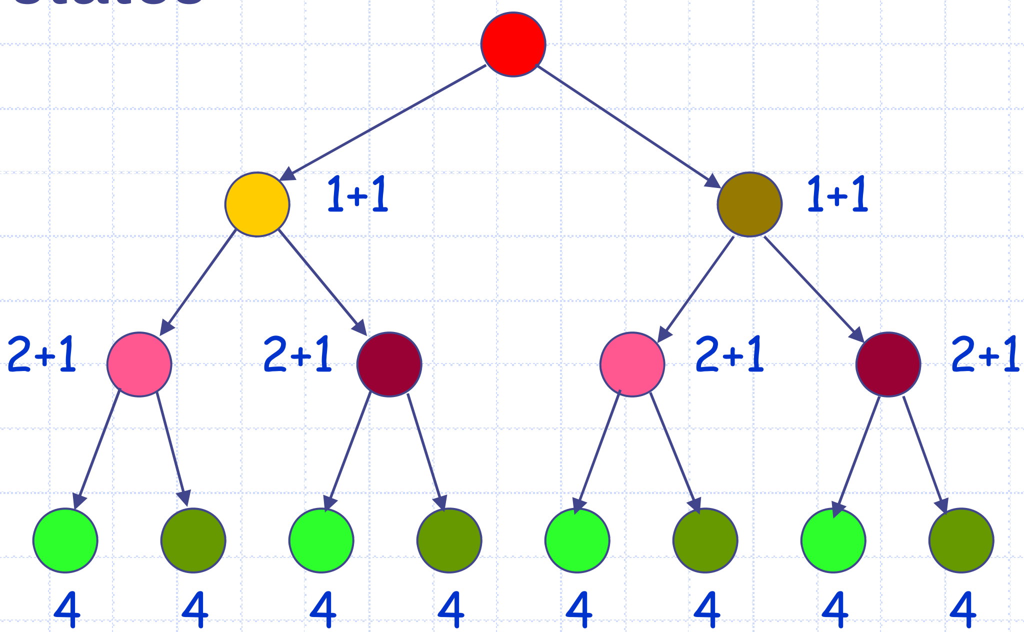
Instead, if we do not discard nodes revisiting states, the search terminates with an optimal solution

But ...

If we do not discard nodes revisiting states, the size of the search tree can be exponential in the number of visited states



2n+1 states



$O(2^n)$ nodes

Handling Revisited States

- It is not harmful to discard a node revisiting a state if the cost of the new path to this state is $\geq$ cost of the previous path

- A* remains optimal, but states can still be re-visited multiple times
  [the size of the search tree can still be exponential in the number of visited states]
- Fortunately, for a large family of admissible heuristics – **consistent heuristics** – there is a much more efficient way to handle revisited states

Consistent Heuristic

A **consistent** heuristic is also **admissible**!

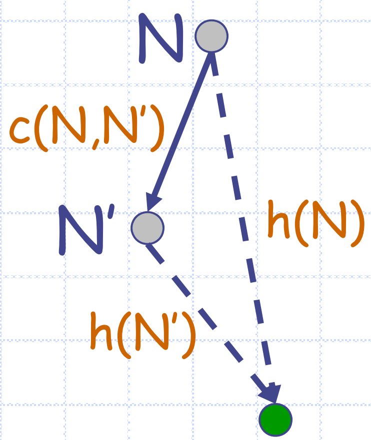
A heuristic **h** is **consistent** (or monotone) if

1) for each node N and each child N' of N:

$$h(N) \leq c(N,N') + h(N')$$

2) for each goal node G:

$$h(G) = 0$$

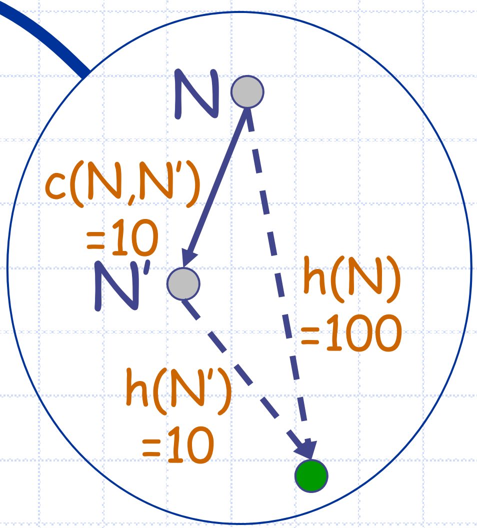


(triangle inequality)

- Intuition: a consistent heuristics becomes more precise as we get deeper in the search tree

Consistency Violation

If h tells that N is 100 units from the goal, then moving from N along an arc costing 10 units should not lead to a node N' that h estimates to be 10 units away from the goal



(triangle inequality)

Admissibility and Consistency

- A consistent heuristic is also admissible
- An admissible heuristic may not be consistent, but many admissible heuristics are consistent

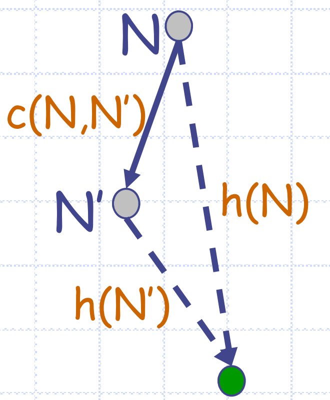
8-Puzzle

5		8
4	2	1
7	3	6

STATE(N)

1	2	3
4	5	6
7	8	

goal

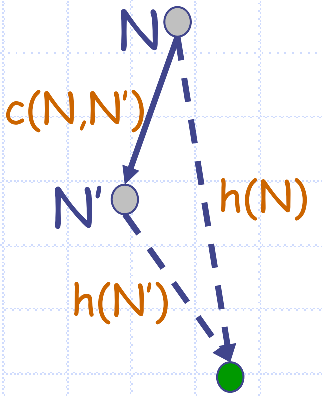


$h(N) \leq c(N,N') + h(N')$

- $h_1(N)$ = number of misplaced tiles
- $h_2(N)$ = sum of the (Manhattan) distances of every tile to its goal position

are both consistent? (why?)

Robot Navigation



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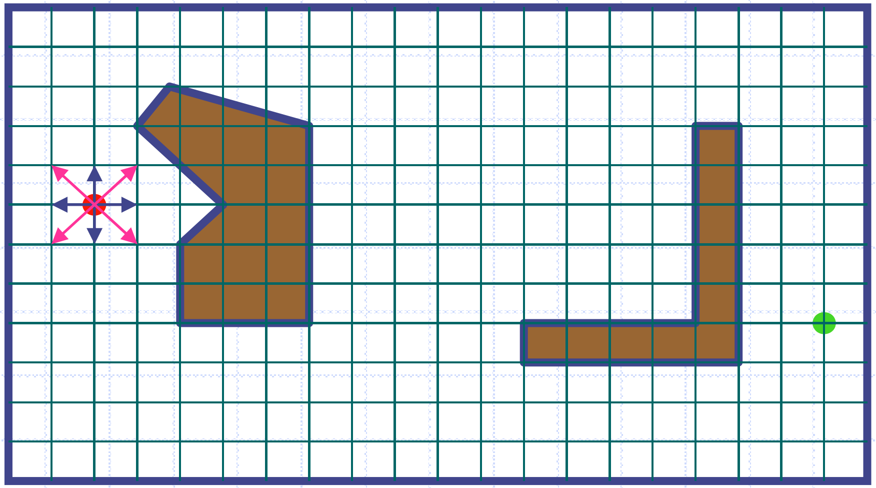


Cost of one horizontal/vertical step = 1

Cost of one diagonal step = $\sqrt{2}$

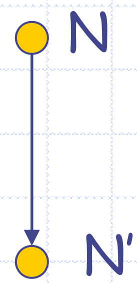
$h_1(N) = \sqrt{(x_N - x_g)^2 + (y_N - y_g)^2}$ is consistent

$h_2(N) = |x_N - x_g| + |y_N - y_g|$ is consistent if moving along diagonals is not allowed, and not consistent otherwise

Result #2

- If h is consistent, then whenever A* expands a node, it has already found an optimal path to this node's state

Proof (1/2)



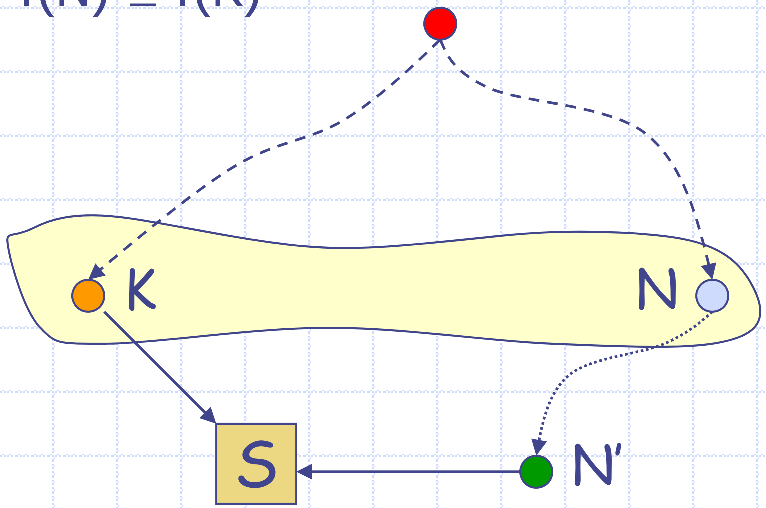
- Consider a node N and its child N'
- Since h is consistent: $h(N) \leq c(N,N') + h(N')$

$$\mathbf{f(N)} = g(N)+h(N) \leq g(N)+c(N,N')+h(N') = \mathbf{f(N')}$$

- So, **f** is non-decreasing along **any** path

Proof (2/2)

- If a node K is selected for expansion, then any other node N in the frontier verifies $f(N) \geq f(K)$



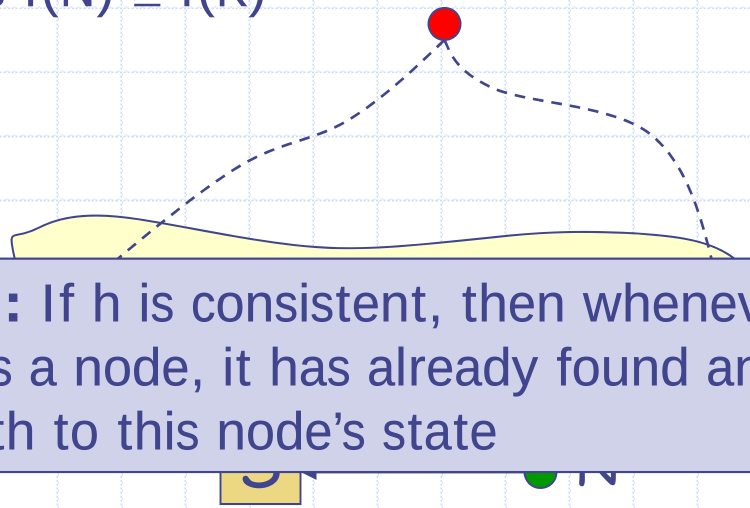
- If one node N lies on another path to the state of K, the cost of this other path is no smaller than that of the path to K:

$$\mathbf{f(N')} = g(N')+h(N') \geq \mathbf{f(N)} \geq \mathbf{f(K)} = g(K)+h(K)$$

- Then because $h(N') = h(K)$, we must have $\mathbf{g(N') \geq g(K)}$

Proof (2/2)

- If a node K is selected for expansion, then any other node N in the frontier verifies f(N) ≥ f(K)



Result #2: If h is consistent, then whenever A* expands a node, it has already found an optimal path to this node's state

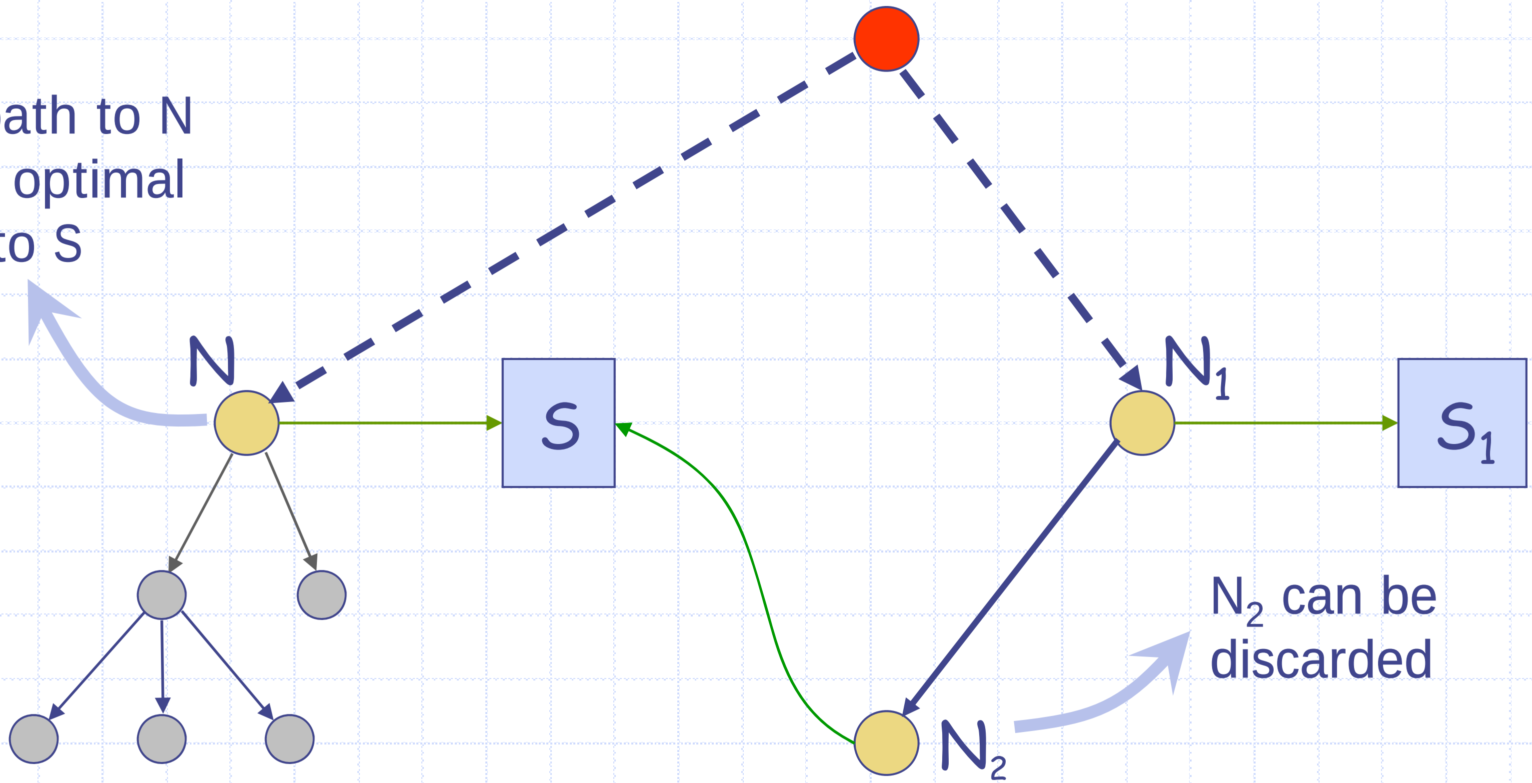
- If one node N lies on another path to the state of K, the cost of this other path is no smaller than that of the path to K:

  f(N') = g(N')+h(N') ≥ **f(N)** ≥ **f(K)** = g(K)+h(K)

- Then because h(N') = h(K), we must have **g(N') ≥ g(K)**

Implication of Result #2

The path to N is the optimal path to S



N_2 can be discarded

Revisited States
with Consistent Heuristic

- When a node is expanded, store its state into CLOSED
- When a new node N is generated:
  - If STATE(N) is in CLOSED, **discard** N
  - If there exists a node N' in the frontier such that STATE(N') = STATE(N), **discard** the node – N or N' – with the largest f (or, equivalently, g)

A* and Consistency

- Is A* with some consistent heuristic all that we need?
- No ! There are some very dumb consistent heuristic functions

For example: $h \equiv 0$

- It is consistent (hence, admissible) !
- A* with $h \equiv 0$ is uniform-cost search
- Breadth-first and uniform-cost are particular cases of A*

Heuristic Accuracy

Let h_1 and h_2 be two consistent heuristics such that for all nodes N:

$$h_1(N) \leq h_2(N)$$

h_2 is said to be **dominate** h_1(or be more accurate or informed)

5		8
4	2	1
7	3	6

STATE(N)

1	2	3
4	5	6
7	8	

Goal state

- $h_1(N)$ = number of misplaced tiles = 6
- $h_2(N)$ = sum of distances of every tile to its goal position = 13
- h_2 is more accurate than h_1

Result #3

- Let h_2 be more accurate than h_1
- Let $\mathbf{A_1*}$ be A* using $\mathbf{h_1}$
  and $\mathbf{A_2*}$ be A* using $\mathbf{h_2}$
- Whenever a solution exists, all the nodes expanded by A_2*
  except possibly for some nodes such that
  $f_1(N) = f_2(N) = C*$ (cost of optimal solution)
  are also expanded by A_1*

Proof

- $C^* = h^*$(initial-node) [cost of optimal solution]
- Every node N such that **$f(N) < C^*$** is eventually expanded.
  No node N such that **$f(N) > C^*$** is ever expanded
- Every node N such that **$h(N) < C^* - g(N)$** is eventually expanded. So, every node N such that $h_2(N) < C^* - g(N)$ is expanded by A_2^*.
  Since **$h_1(N) \leq h_2(N) < C^* - g(N)$** , N is also expanded by A_1^*
- If there are several nodes N such that $f_1(N) = f_2(N) = C^*$ (such nodes include the optimal goal nodes, if there exists a solution), A_1^* and A_2^* may or may not expand them in the same order (until one goal node is expanded)

Effective Branching Factor

- It is used as a measure the effectiveness of a heuristic
- Let **n** be the total number of nodes expanded by A* for a particular problem and **d** the depth of the solution
- The effective branching factor **b*** is defined by

$$n = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$$

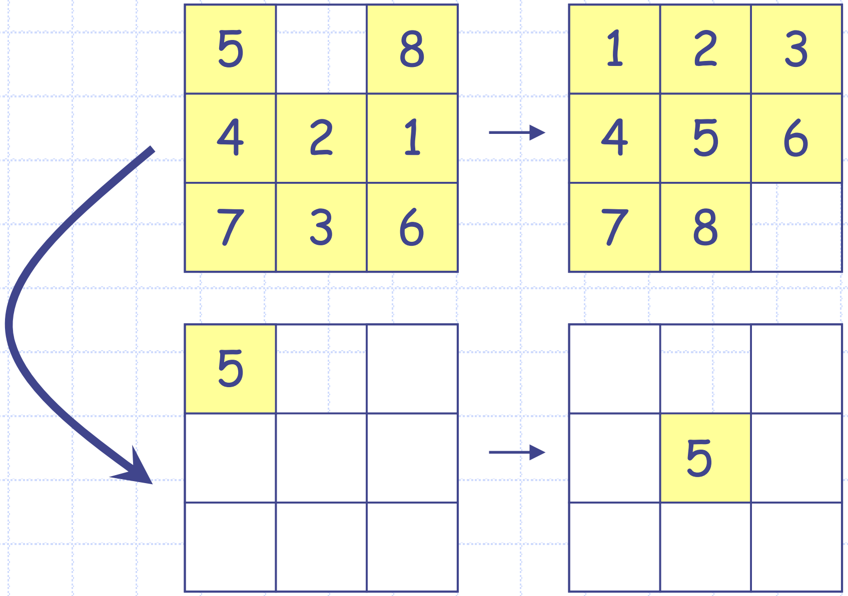
Experimental Results

- 8-puzzle with:
  - h_1 = number of misplaced tiles
  - h_2 = sum of distances of tiles to their goal positions
- Random generation of many problem instances
- Average effective branching factors (number of expanded nodes):

d	IDS	A_1*	A_2*
2	2.45	1.79	1.79
6	2.73	1.34	1.30
12	2.78 (3,644,035)	1.42 (227)	1.24 (73)
16	--	1.45	1.25
20	--	1.47	1.27
24	--	1.48 (39,135)	1.26 (1,641)

How to create good heuristics?

- By solving **relaxed** problems at each node
- In the 8-puzzle, the sum of the distances of each tile to its goal position (h_2) corresponds to solving 8 simple problems:



d_i is the length of the shortest path to move tile i to its goal position, ignoring the other tiles, e.g., $d_5 = 2$

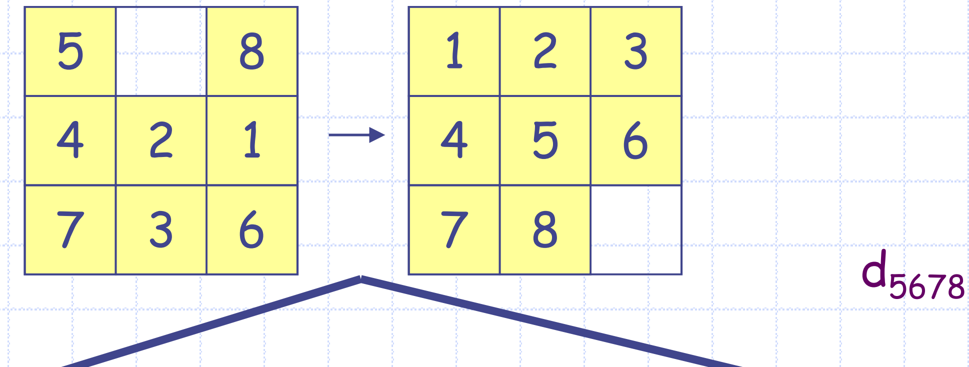
$h_2 = \Sigma_{i=1,...8}\ d_i$

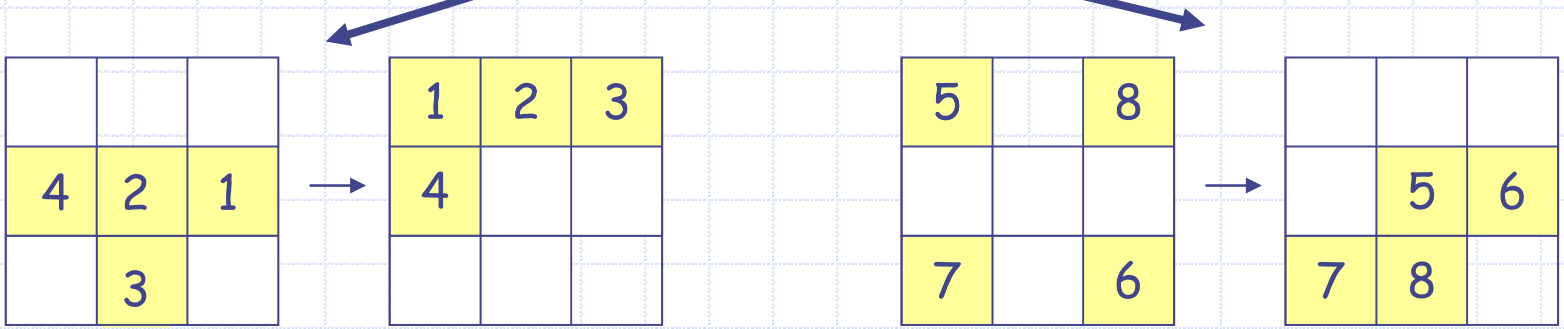
- It ignores negative interactions among tiles

Can we do better?

- For example, we could consider two more complex relaxed problems:

d_{1234} = length of the shortest path to move tiles 1, 2, 3, and 4 to their goal positions, ignoring the other tiles





- → $h = d_{1234} + d_{5678}$ [disjoint pattern heuristic]
- How to compute d_{1234} and d_{5678}?

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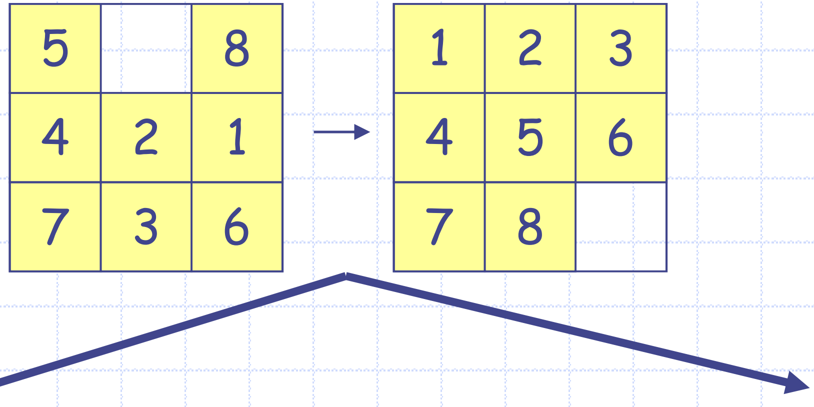
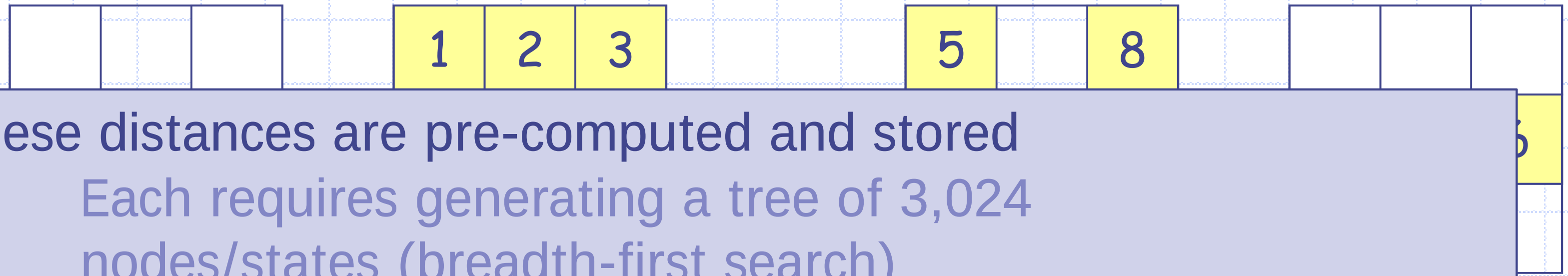


d_{5678}



- These distances are pre-computed and stored
  - Each requires generating a tree of 3,024 nodes/states (breadth-first search)
- Several order-of-magnitude speedups for the 15- and 24-puzzle

- How to compute d_{1234} and d_{5678}?

Note on

Completeness and Optimality

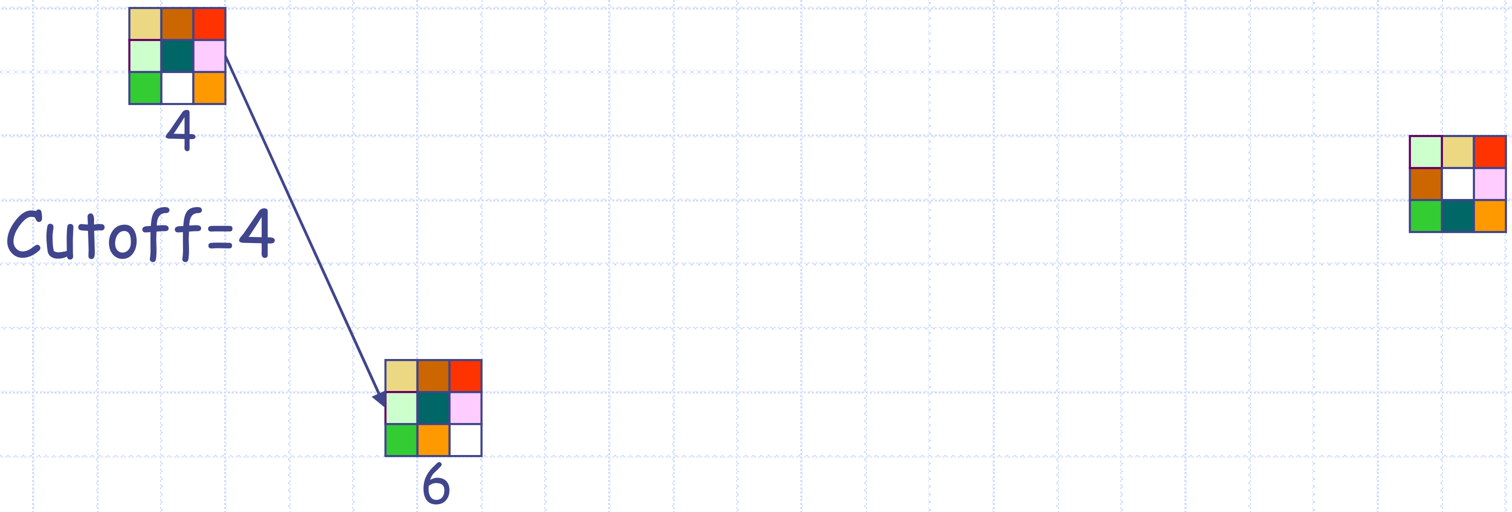
- A* with a consistent heuristic function has nice properties: completeness, optimality, no need to revisit states
- Theoretical completeness does not mean "practical" completeness if you must wait too long to get a solution (e.g. time limit issue)
- So, if one can't design an accurate consistent heuristic, it may be better to settle for a non-admissible heuristic that "works well in practice", even through completeness and optimality are no longer guaranteed

Iterative Deepening A* (IDA*)

- Idea: Reduce memory requirement of A* by applying **cutoff** on values of **f**
- Consistent heuristic function h
- Algorithm IDA*:
  - Initialize cutoff to f(initial-node)
  - Repeat:
    - Perform depth-first search by expanding all nodes N such that $f(N) \leq$ cutoff
    - Reset cutoff to smallest value f of non-expanded (leaf) nodes

8-Puzzle

f(N) = g(N) + h(N)
with h(N) = number of misplaced tiles



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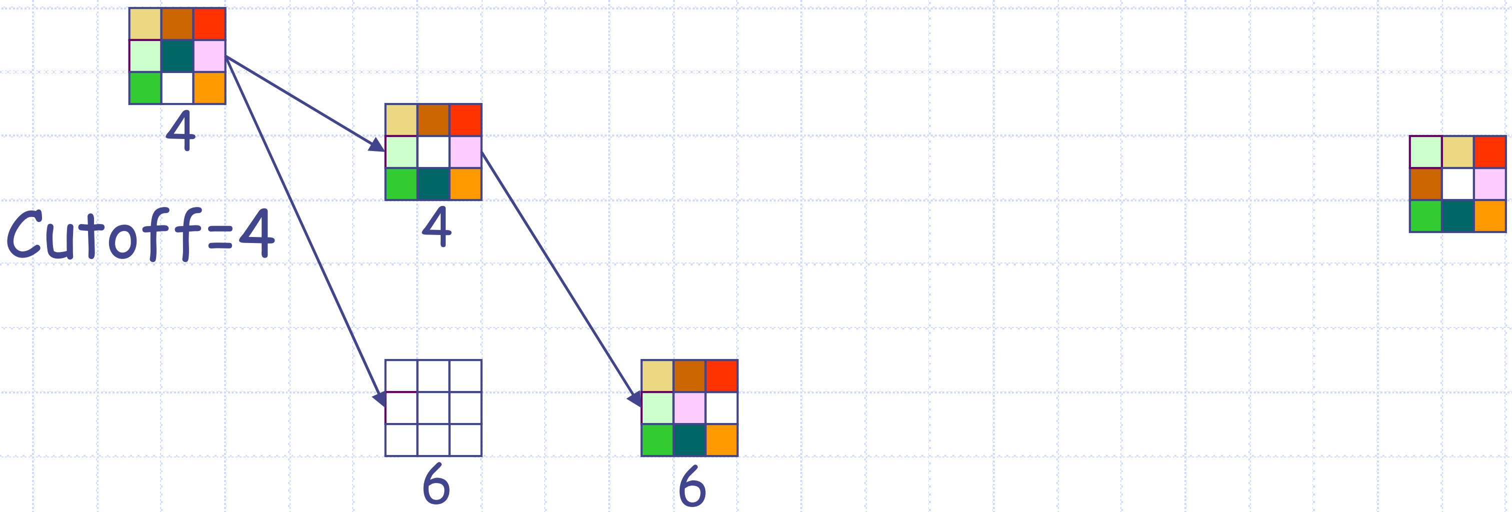

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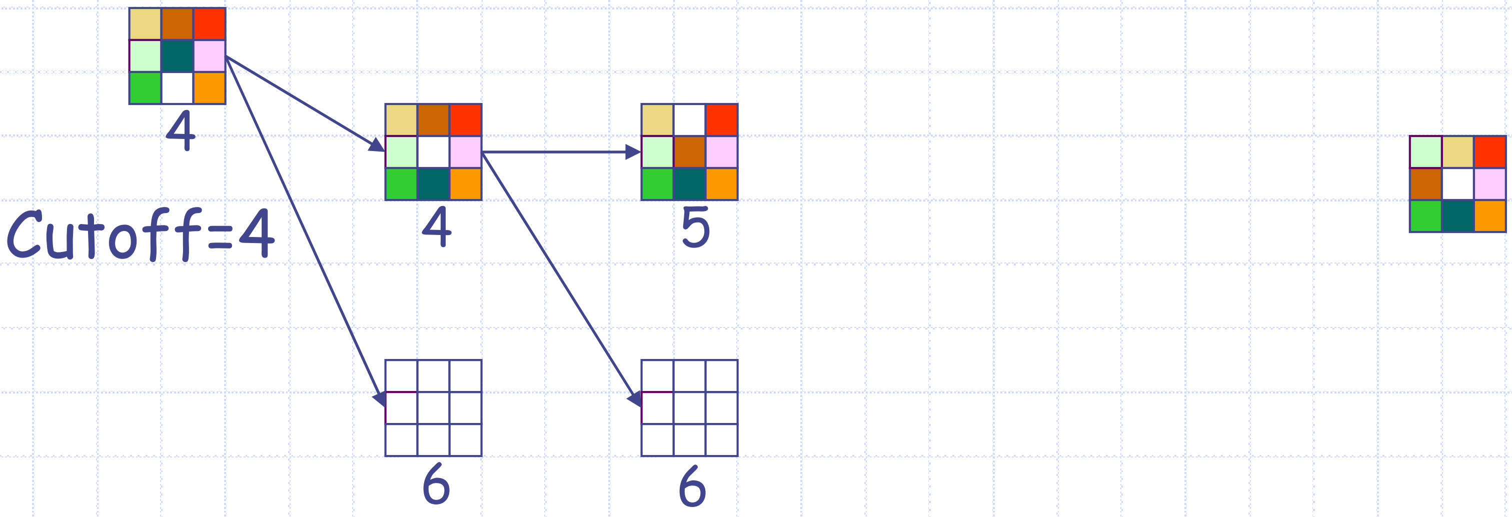

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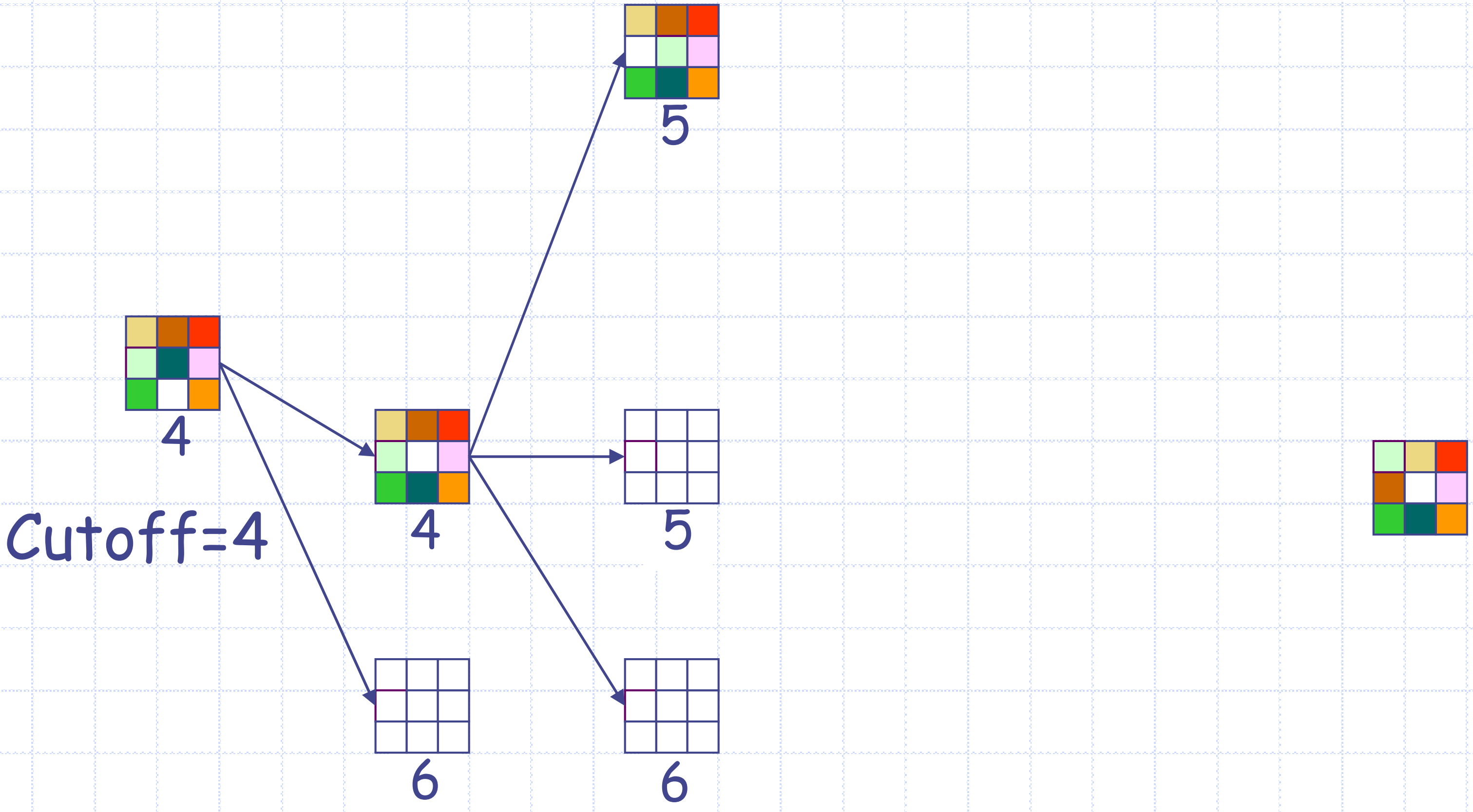

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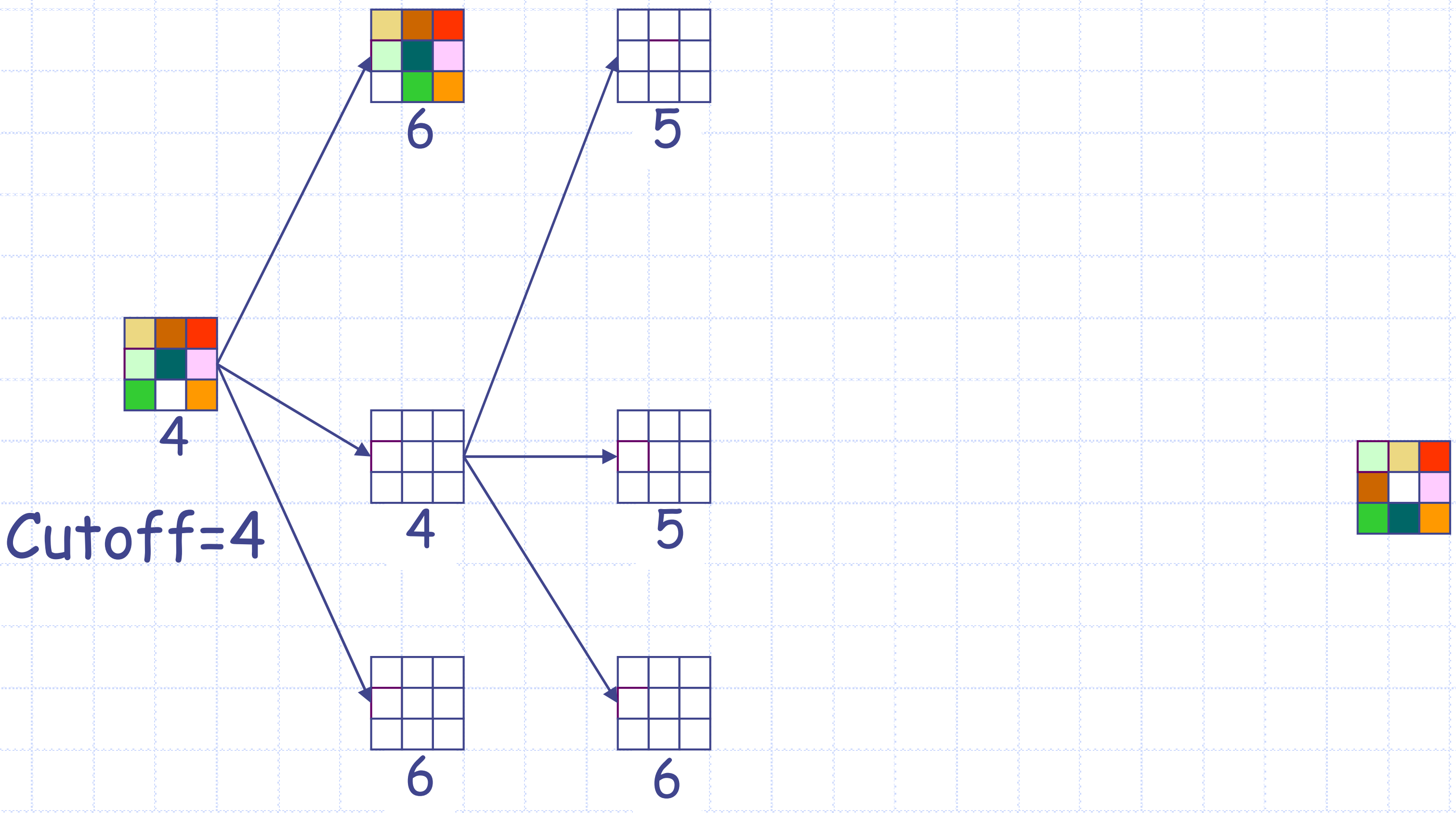

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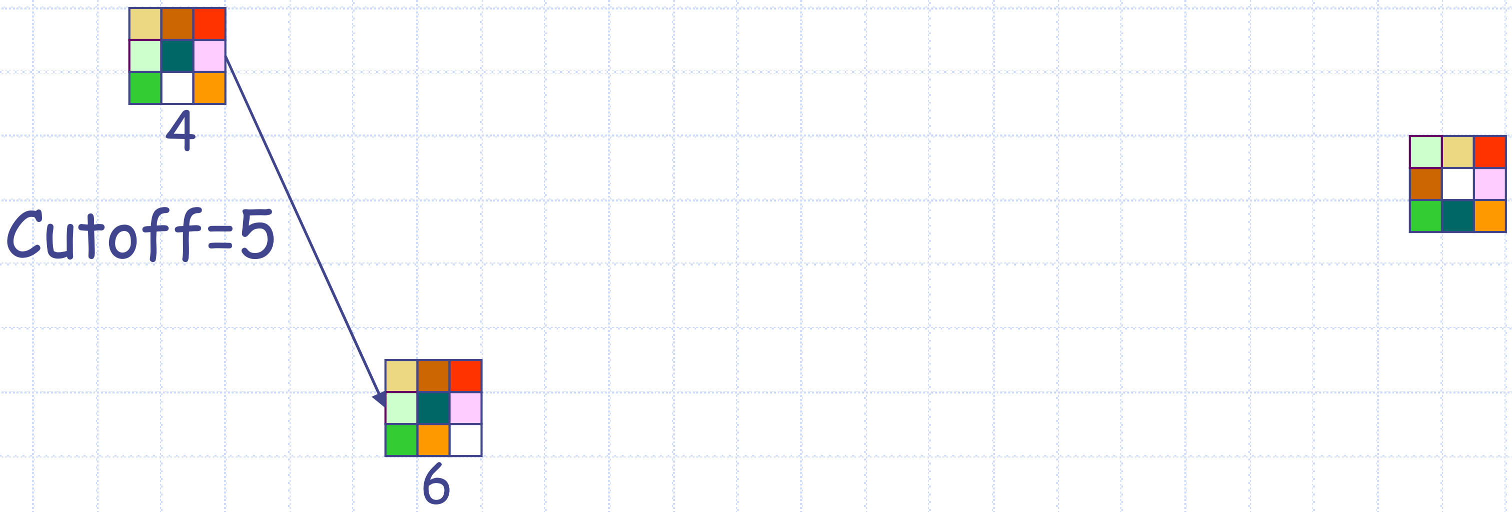

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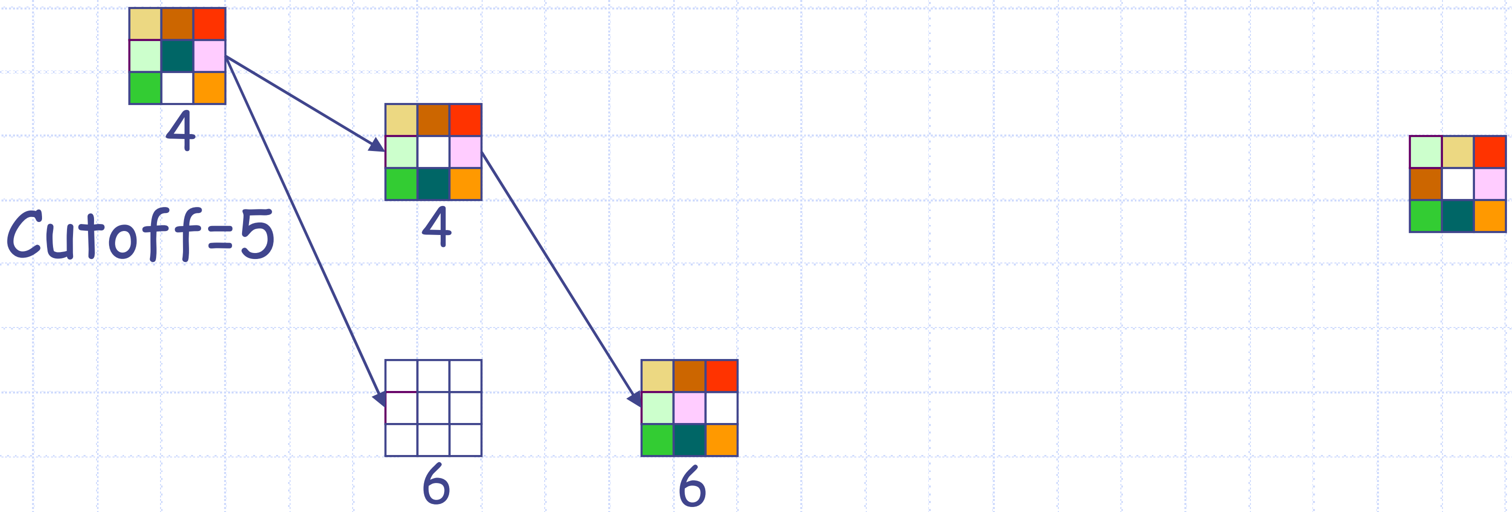

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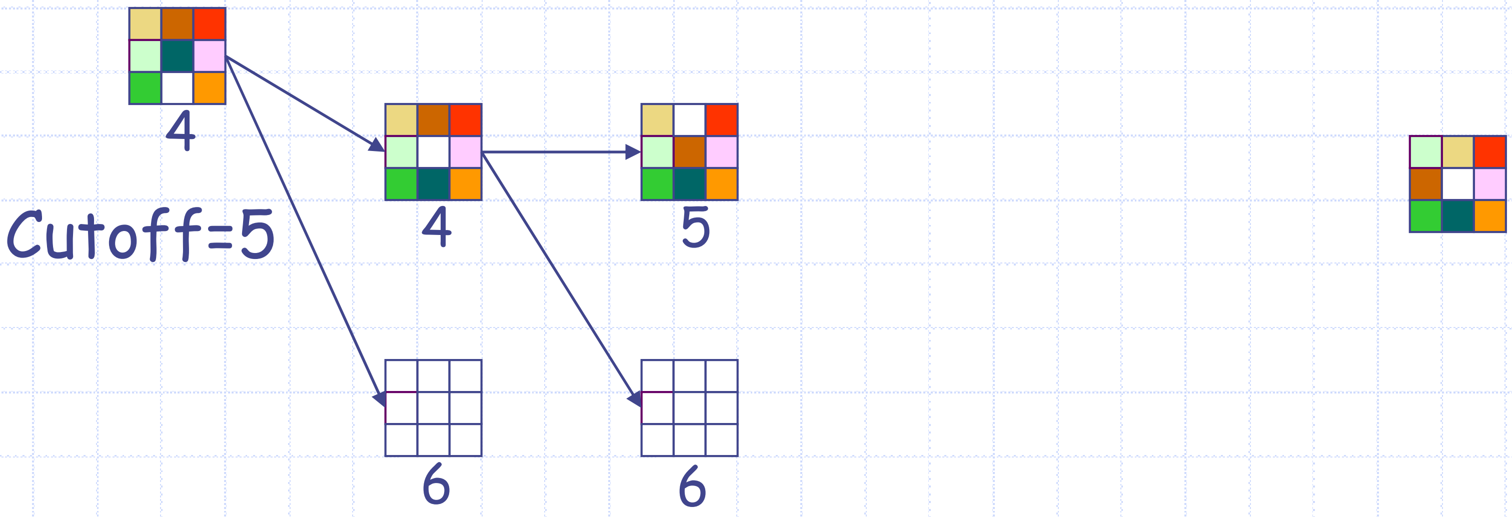

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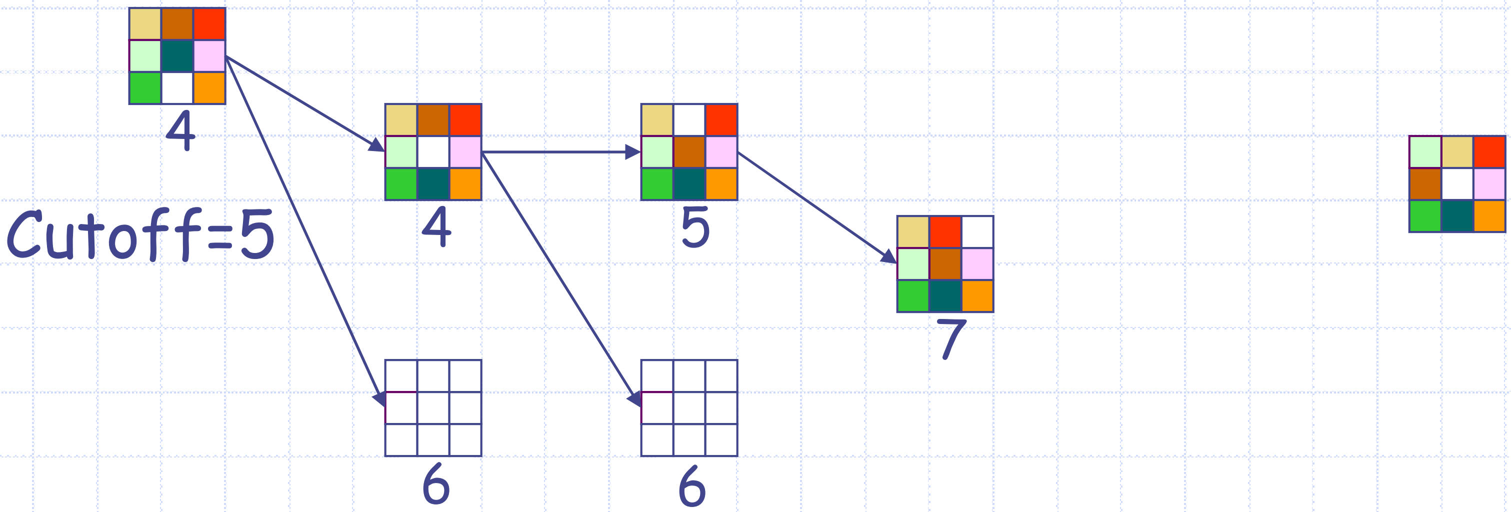

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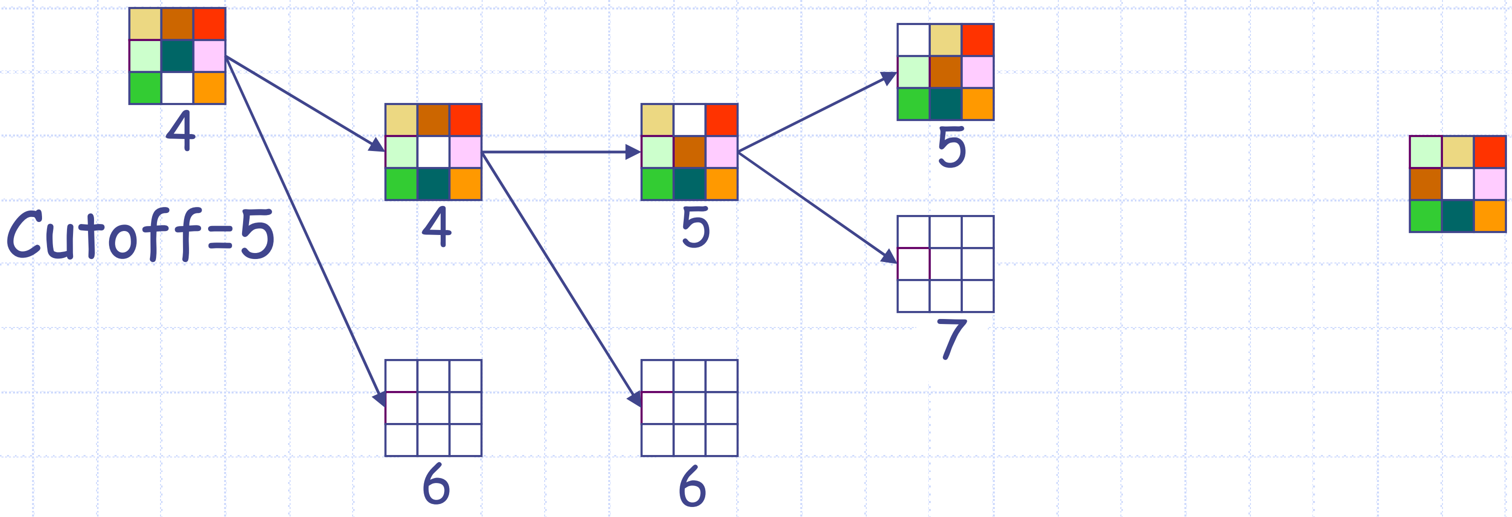

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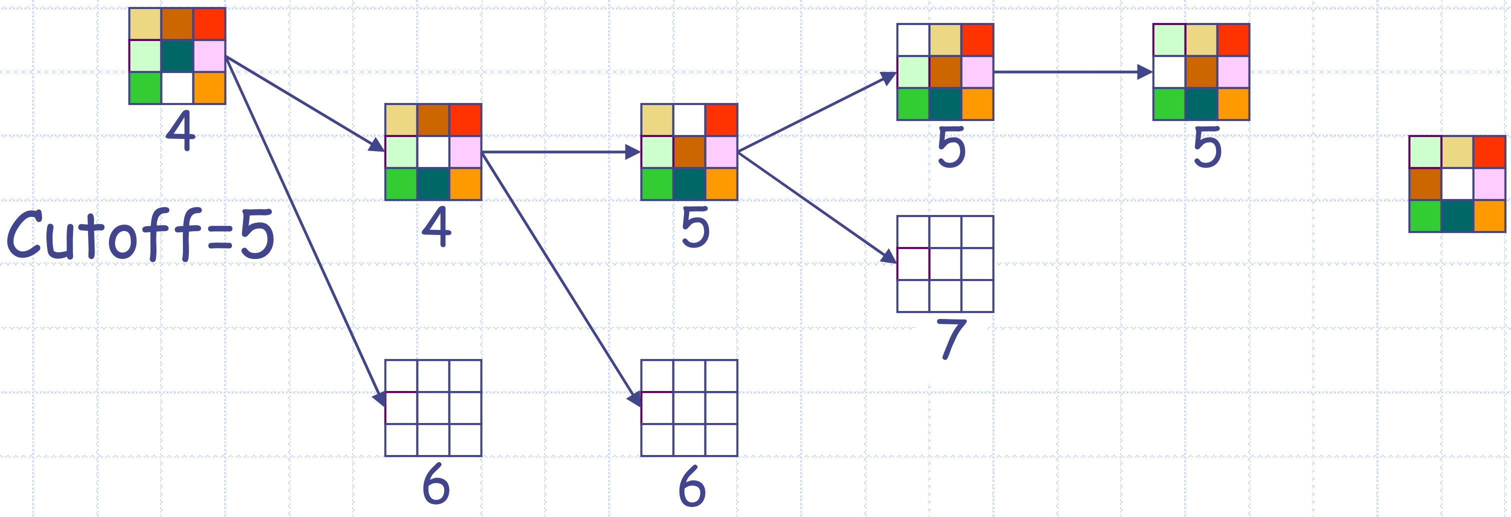

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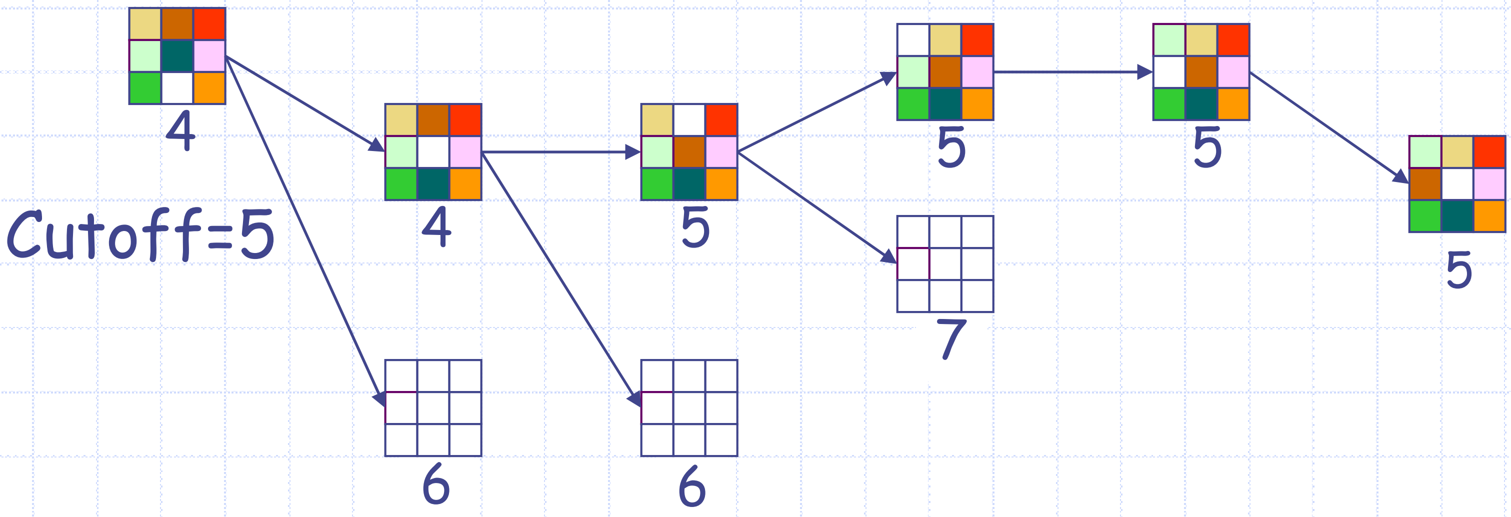


Advantages/Drawbacks of IDA*

- Advantages:
  - Still complete and optimal
  - Requires less memory than A*
  - Avoid the overhead to sort the frontier
- Drawbacks:
  - Can't avoid revisiting states not on the current path
  - Available memory is poorly used (→ memory-bounded search)

Local Search

- Light-memory search method
- No search tree; only the current state is represented!
- Only applicable to problems where the path is irrelevant (e.g., 8-queen), unless the path is encoded in the state
- Many similarities with optimization techniques

Steepest Descent

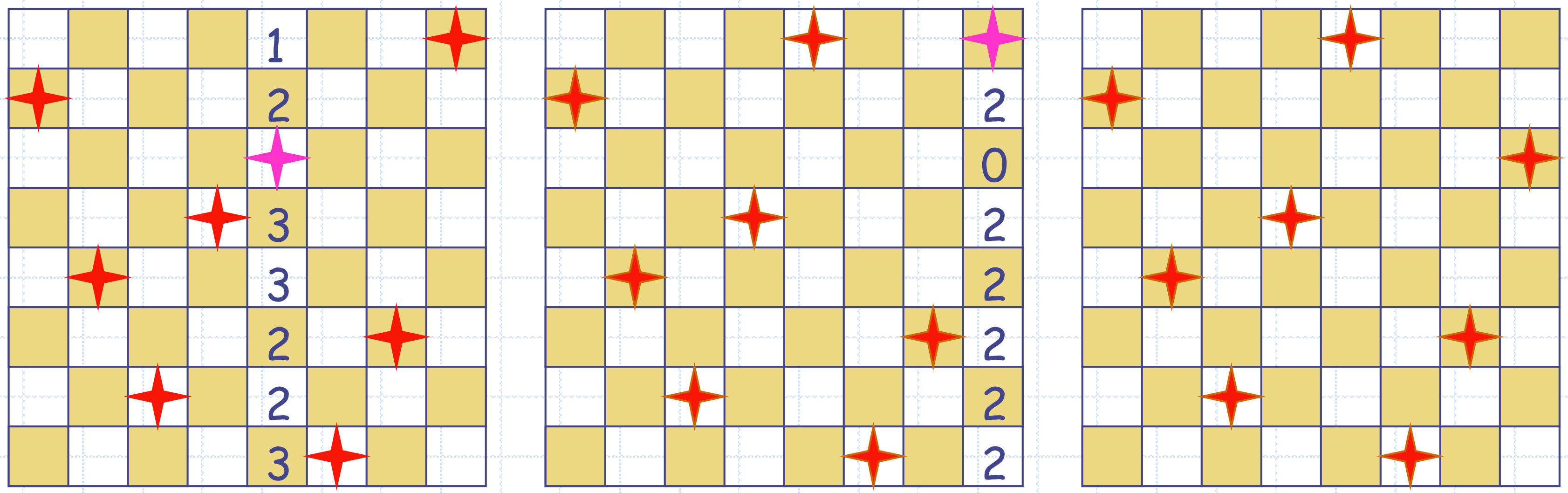
1) S ← initial state
2) Repeat:
   a) $S' \leftarrow \arg\min_{S' \in SUCCESSORS(S)} \{h(S')\}$
   b) if GOAL?(S') return S'
   c) if $h(S') < h(S)$ then $S \leftarrow S'$ else return failure

Similar to:

- hill climbing with –h
- gradient descent over continuous space

Application: 8-Queen

1) Pick an initial state S at random with one queen in each column
2) Repeat k times:
   a) If GOAL?(S) then return S
   b) Pick an attacked queen Q at random
   c) Move Q in its column to minimize the number of attacking queens → new S [min-conflicts heuristic]
3) Return failure



Application: 8-Queen

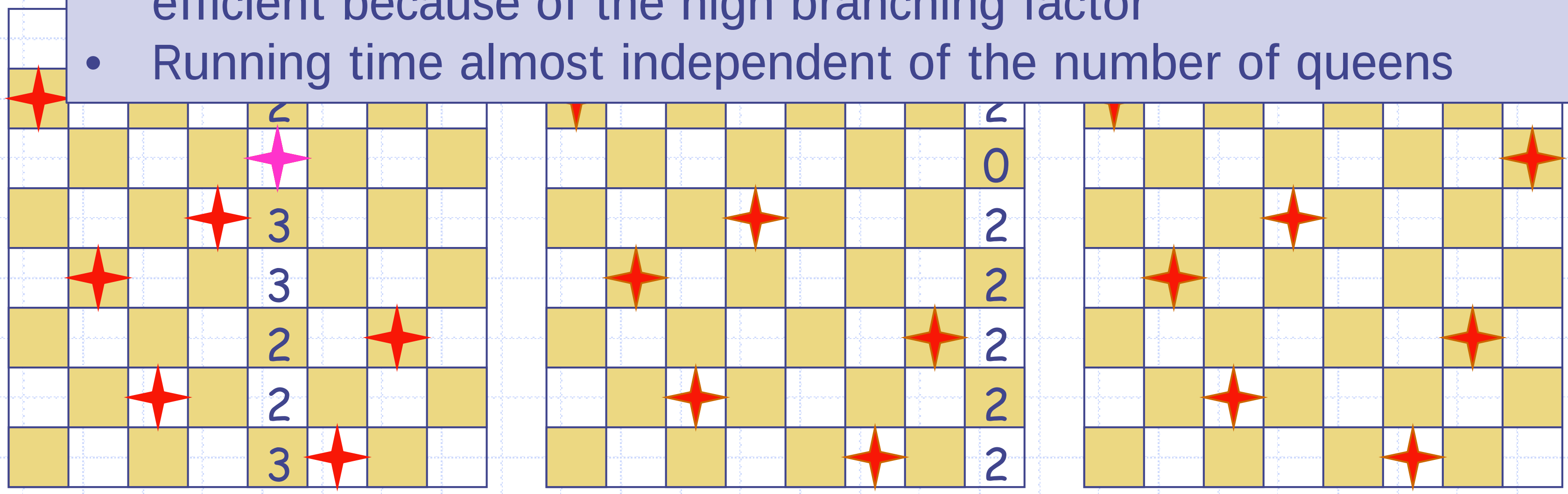
Repeat n times:

1) Pick an initial state S at random with one queen in each column

2) Repeat k times:

Why does it work ?

- There are many goal states that are well-distributed over the state space
- If no solution has been found after a few steps, it's better to start it all over again. Building a search tree would be much less efficient because of the high branching factor
- Running time almost independent of the number of queens



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may easily get stuck in local minima

→ Random restart (as in n-queen example)

→ Monte Carlo descent

Monte Carlo Descent

1) S ← initial state
2) Repeat k times:
   a) If GOAL?(S) then return S
   b) S' ← successor of S picked at random
   c) if h(S') ≤ h(S) then S ← S'
   d) else
      - $\Delta h = h(S')-h(S)$
      - with probability ~ $\exp(-\Delta h/T)$, where T is called the "temperature", do: S ← S' [Metropolis criterion]
3) Return failure

Simulated annealing lowers T over the k iterations.

It starts with a large T and slowly decreases T

“Parallel” Local Search

- They perform several local searches concurrently, but not independently:
  - Beam search
  - Genetic algorithms

When Use Search Techniques?

- The search space is small, and
  - No other technique is available, or
  - Developing a more efficient technique is not worth the effort
- The search space is large, and
  - No other available technique is available, and
  - There exist “good” heuristics