Action Planning

(Where logic-based representation of knowledge makes search problems more interesting)

R&N: Chap. 11, Sect. 11.1–4

Slides from Jean-Claude Latombe at Stanford University
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- The goal of action planning is to choose actions and ordering relations among these actions to achieve specified goals
- Search-based problem solving applied to 8-puzzle was one example of planning, but our description of this problem used specific data structures and functions
- Here, we will develop a non-specific, logic-based language to represent knowledge about actions, states, and goals, and we will study how search algorithms can exploit this representation

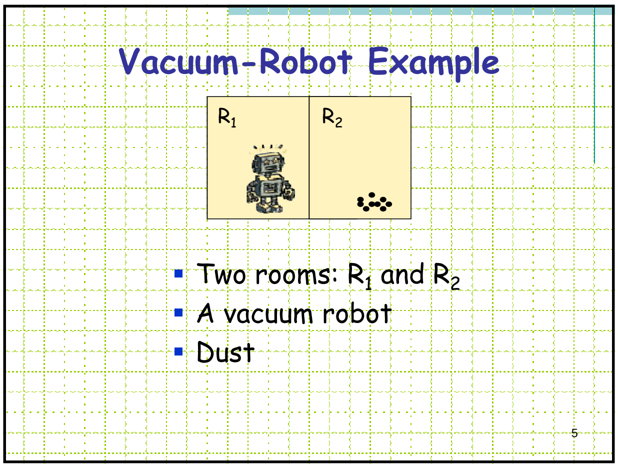
Knowledge Representation Tradeoff

- Expressiveness vs. computational efficiency
- STRIPS: a simple, still reasonably expressive planning language based on propositional logic
  1) Examples of planning problems in STRIPS
  2) Planning methods
  3) Extensions of STRIPS
- Like programming, knowledge representation is still an art

SHAKEY
the robot

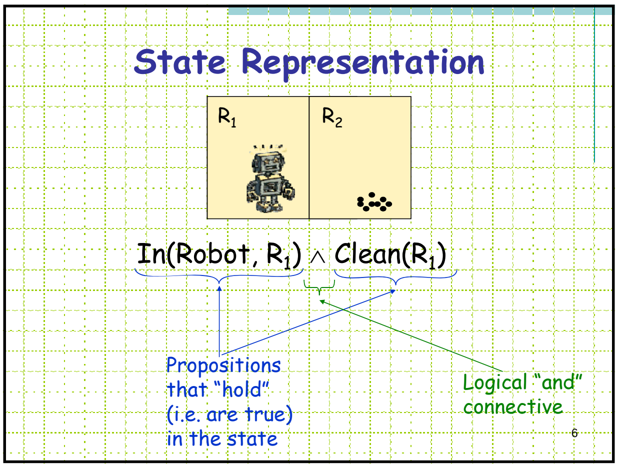
STRIPS Language through Examples

Vacuum-Robot Example



- Two rooms: R_1 and R_2
- A vacuum robot
- Dust

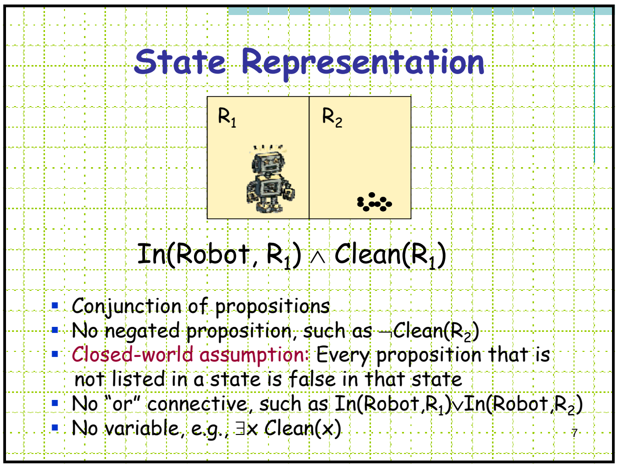
State Representation



$$\underbrace{In(Robot, R_1)}\ \underbrace{\wedge}\ \underbrace{Clean(R_1)}$$

Propositions that "hold" (i.e. are true) in the state

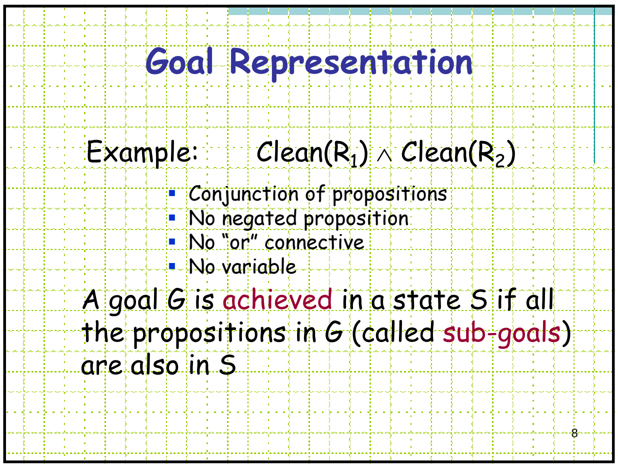
Logical "and" connective



State Representation

$In(Robot, R_1) \wedge Clean(R_1)$

- Conjunction of propositions
- No negated proposition, such as $\neg Clean(R_2)$
- Closed-world assumption: Every proposition that is not listed in a state is false in that state
- No "or" connective, such as $In(Robot,R_1) \vee In(Robot,R_2)$
- No variable, e.g., $\exists x\ Clean(x)$

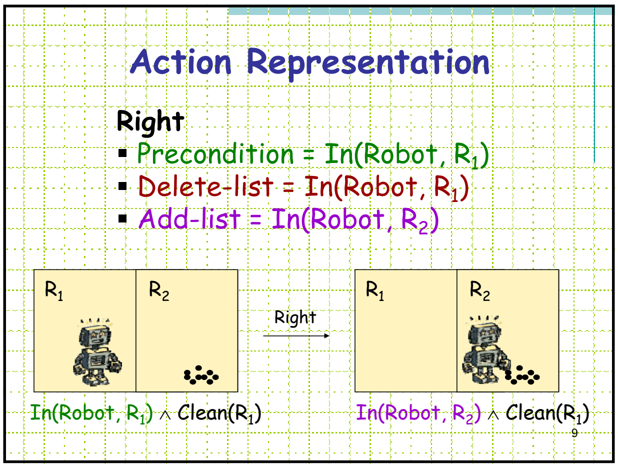


Goal Representation

Example: $Clean(R_1) \wedge Clean(R_2)$

- Conjunction of propositions
- No negated proposition
- No "or" connective
- No variable

A goal G is achieved in a state S if all the propositions in G (called sub-goals) are also in S



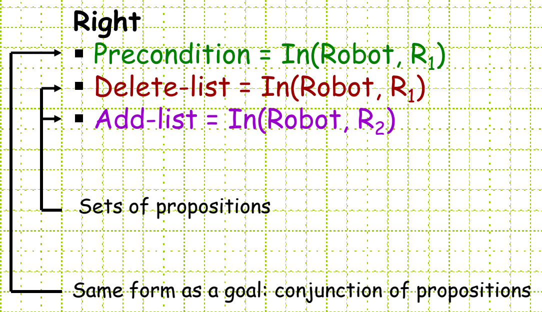
Action Representation

Right

- Precondition = $In(Robot, R_1)$
- Delete-list = $In(Robot, R_1)$
- Add-list = $In(Robot, R_2)$

$In(Robot, R_1) \wedge Clean(R_1)$ $In(Robot, R_2) \wedge Clean(R_1)$

Action Representation



Right

- Precondition = In(Robot, R_1)
- Delete-list = In(Robot, R_1)
- Add-list = In(Robot, R_2)

Sets of propositions

Same form as a goal: conjunction of propositions

Action Representation

Right

- Precondition = In(Robot, R_1)
- Delete-list = In(Robot, R_1)
- Add-list = In(Robot, R_2)

- An action A is applicable to a state S if the propositions in its precondition are all in S
- The application of A to S is a new state obtained by deleting the propositions in the delete list from S and adding those in the add list

Other Actions

Left

- P = In(Robot, R_2)
- D = In(Robot, R_2)
- A = In(Robot, R_1)

Suck(R_1)

- P = In(Robot, R_1)
- D = $\varnothing$ [empty list]
- A = Clean(R_1)

Suck(R_2)

- P = In(Robot, R_2)
- D = $\varnothing$ [empty list]
- A = Clean(R_2)

Other Actions

Left

- P = In(Robot, R_2)
- D = In(Robot, R_2)
- A = In(Robot, R_1)

Suck(r)

- P = In(Robot, r)
- D = ∅ [empty list]
- A = Clean(r)

Action Schema

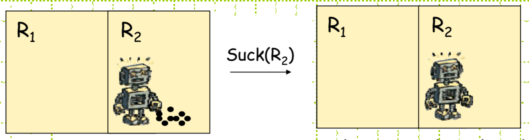
It describes several actions, here: Suck(R_1) and Suck(R_2)

Parameter that will get "instantiated" by matching the precondition against a state

Suck(r)

- P = In(Robot, r)
- D = ∅
- A = Clean(r)

Action Schema



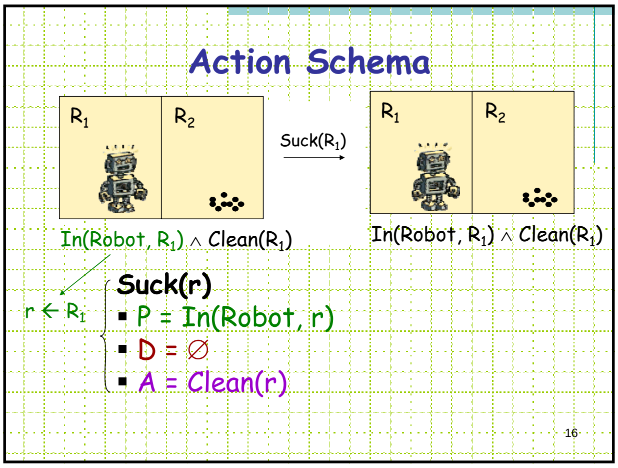
In(Robot, R_2) ∧ Clean(R_1)

In(Robot, R_2) ∧ Clean(R_1) ∧ Clean(R_2)

r ← R_2

Suck(r)

- P = In(Robot, r)
- D = ∅
- A = Clean(r)



Action Schema

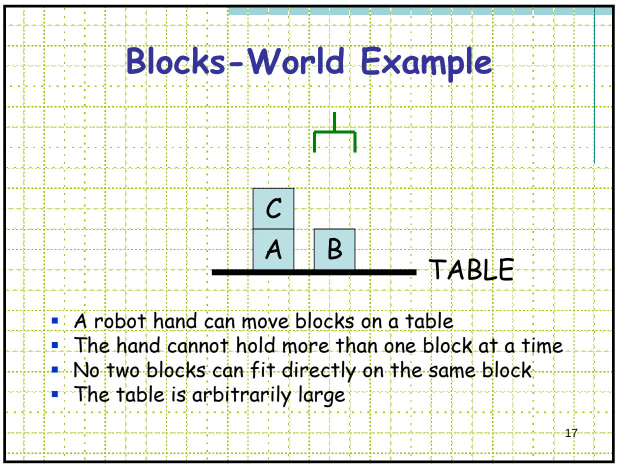
In(Robot, R_1) $\wedge$ Clean(R_1)

In(Robot, R_1) $\wedge$ Clean(R_1)

$r \leftarrow R_1$

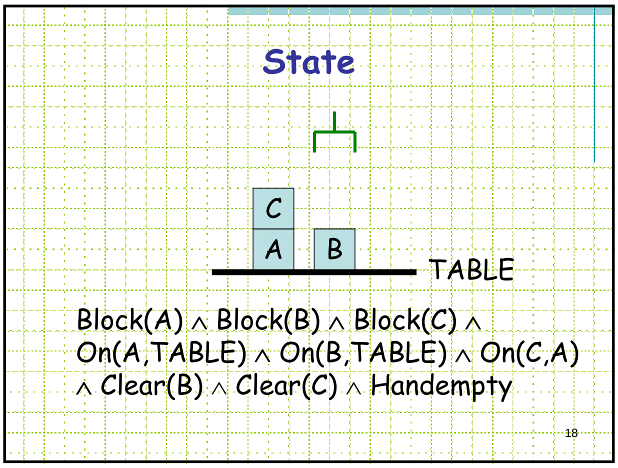
Suck(r)

- P = In(Robot, r)
- D = $\varnothing$
- A = Clean(r)



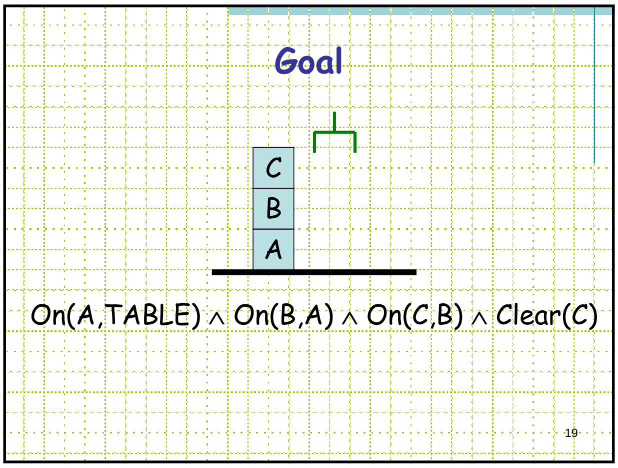
Blocks-World Example

- A robot hand can move blocks on a table
- The hand cannot hold more than one block at a time
- No two blocks can fit directly on the same block
- The table is arbitrarily large



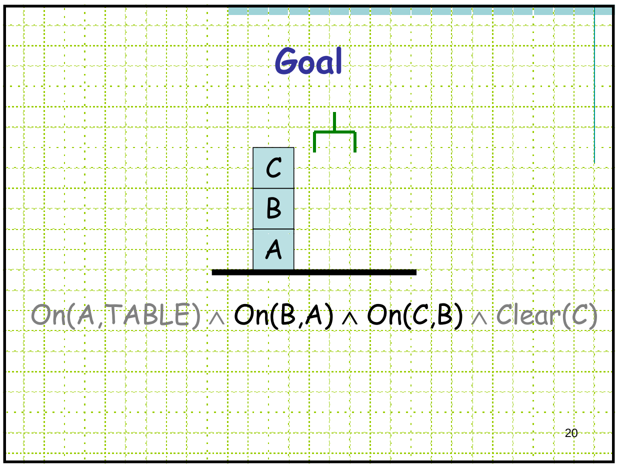
State

Block(A) $\wedge$ Block(B) $\wedge$ Block(C) $\wedge$
On(A,TABLE) $\wedge$ On(B,TABLE) $\wedge$ On(C,A)
$\wedge$ Clear(B) $\wedge$ Clear(C) $\wedge$ Handempty



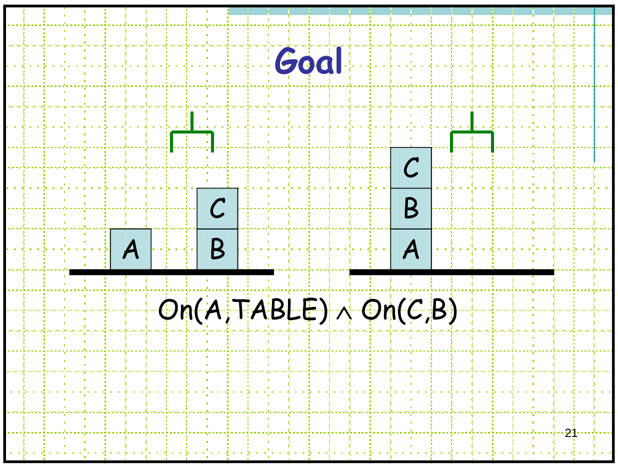
Goal

On(A,TABLE) ∧ On(B,A) ∧ On(C,B) ∧ Clear(C)



Goal

On(A,TABLE) ∧ On(B,A) ∧ On(C,B) ∧ Clear(C)

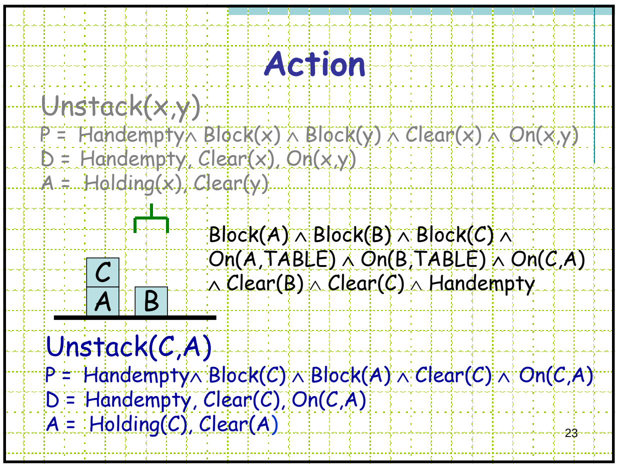


Goal

On(A,TABLE) ∧ On(C,B)

Action

Unstack(x,y)
P = Handempty∧ Block(x) ∧ Block(y) ∧ Clear(x) ∧ On(x,y)
D = Handempty, Clear(x), On(x,y)
A = Holding(x), Clear(y)

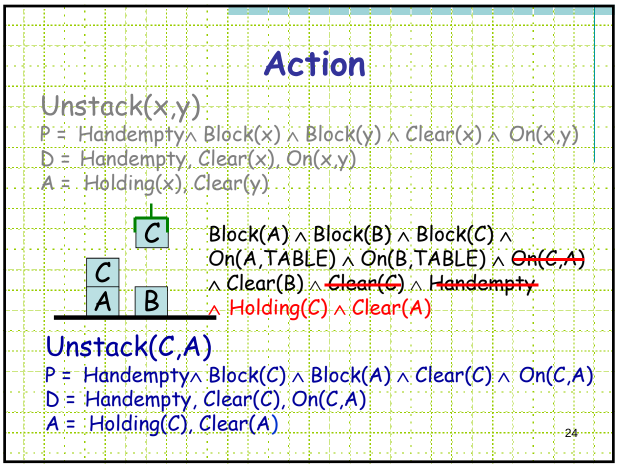


Action

Unstack(x,y)
P = Handempty∧ Block(x) ∧ Block(y) ∧ Clear(x) ∧ On(x,y)
D = Handempty, Clear(x), On(x,y)
A = Holding(x), Clear(y)

Block(A) ∧ Block(B) ∧ Block(C) ∧
On(A,TABLE) ∧ On(B,TABLE) ∧ On(C,A)
∧ Clear(B) ∧ Clear(C) ∧ Handempty

Unstack(C,A)
P = Handempty∧ Block(C) ∧ Block(A) ∧ Clear(C) ∧ On(C,A)
D = Handempty, Clear(C), On(C,A)
A = Holding(C), Clear(A)

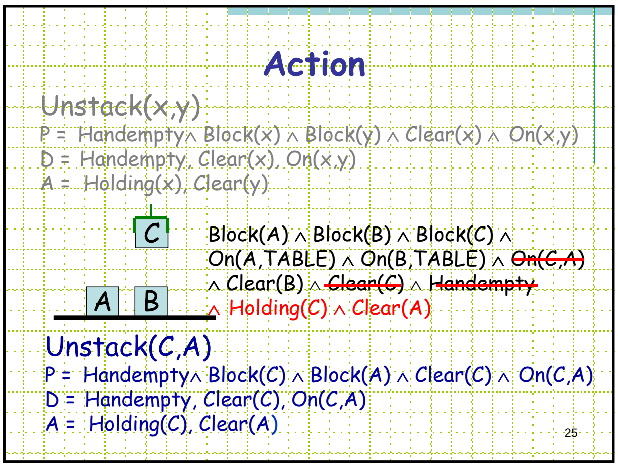


Action

Unstack(x,y)
P = Handempty∧ Block(x) ∧ Block(y) ∧ Clear(x) ∧ On(x,y)
D = Handempty, Clear(x), On(x,y)
A = Holding(x), Clear(y)

Block(A) ∧ Block(B) ∧ Block(C) ∧
On(A,TABLE) ∧ On(B,TABLE) ∧ ~~On(C,A)~~
∧ Clear(B) ∧ ~~Clear(C)~~ ∧ ~~Handempty~~
∧ Holding(C) ∧ Clear(A)

Unstack(C,A)
P = Handempty∧ Block(C) ∧ Block(A) ∧ Clear(C) ∧ On(C,A)
D = Handempty, Clear(C), On(C,A)
A = Holding(C), Clear(A)

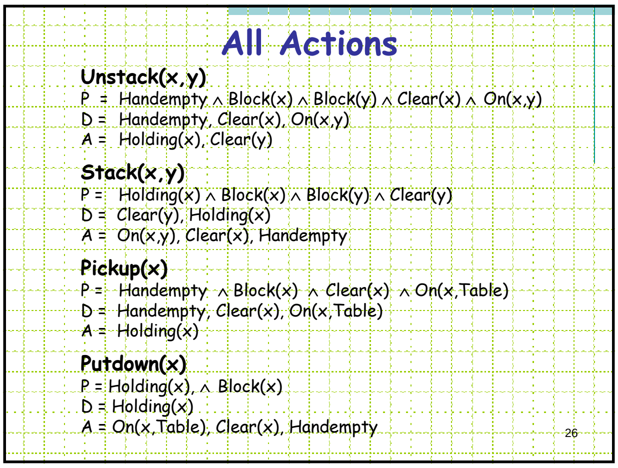


Action

Unstack(x,y)
P = Handempty∧ Block(x) ∧ Block(y) ∧ Clear(x) ∧ On(x,y)
D = Handempty, Clear(x), On(x,y)
A = Holding(x), Clear(y)

Block(A) ∧ Block(B) ∧ Block(C) ∧
On(A,TABLE) ∧ On(B,TABLE) ∧ ~~On(C,A)~~
∧ Clear(B) ∧ ~~Clear(C)~~ ∧ ~~Handempty~~
∧ Holding(C) ∧ Clear(A)

Unstack(C,A)
P = Handempty∧ Block(C) ∧ Block(A) ∧ Clear(C) ∧ On(C,A)
D = Handempty, Clear(C), On(C,A)
A = Holding(C), Clear(A)



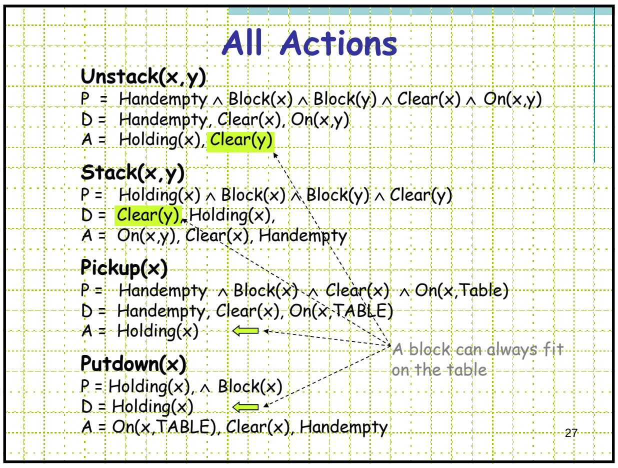
All Actions

Unstack(x,y)
P = Handempty ∧ Block(x) ∧ Block(y) ∧ Clear(x) ∧ On(x,y)
D = Handempty, Clear(x), On(x,y)
A = Holding(x), Clear(y)

Stack(x,y)
P = Holding(x) ∧ Block(x) ∧ Block(y) ∧ Clear(y)
D = Clear(y), Holding(x)
A = On(x,y), Clear(x), Handempty

Pickup(x)
P = Handempty ∧ Block(x) ∧ Clear(x) ∧ On(x,Table)
D = Handempty, Clear(x), On(x,Table)
A = Holding(x)

Putdown(x)
P = Holding(x), ∧ Block(x)
D = Holding(x)
A = On(x,Table), Clear(x), Handempty



All Actions

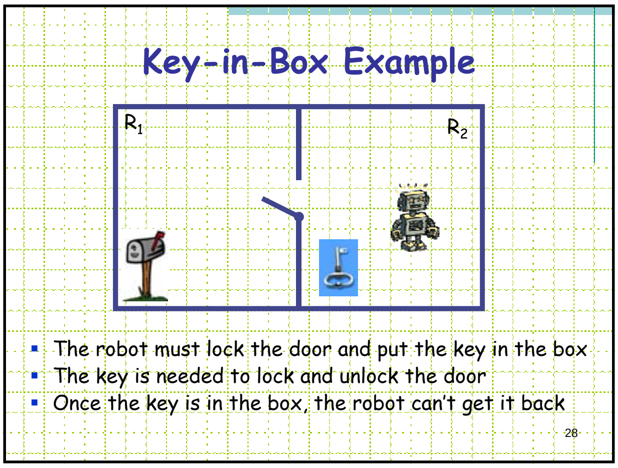
Unstack(x,y)
P = Handempty ∧ Block(x) ∧ Block(y) ∧ Clear(x) ∧ On(x,y)
D = Handempty, Clear(x), On(x,y)
A = Holding(x), Clear(y)

Stack(x,y)
P = Holding(x) ∧ Block(x) ∧ Block(y) ∧ Clear(y)
D = Clear(y), Holding(x),
A = On(x,y), Clear(x), Handempty

Pickup(x)
P = Handempty ∧ Block(x) ∧ Clear(x) ∧ On(x,Table)
D = Handempty, Clear(x), On(x,TABLE)
A = Holding(x)

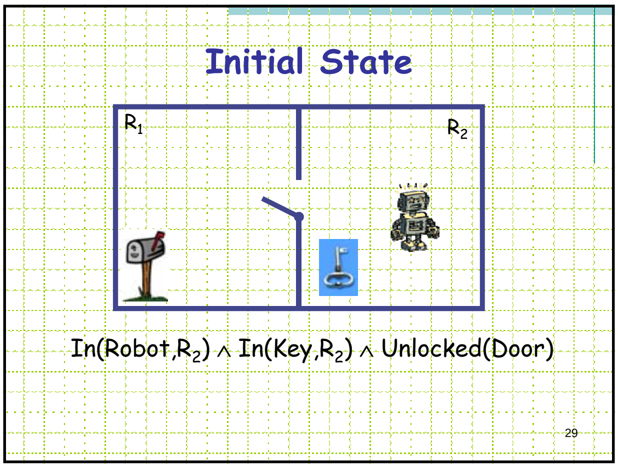
A block can always fit on the table

Putdown(x)
P = Holding(x), ∧ Block(x)
D = Holding(x)
A = On(x,TABLE), Clear(x), Handempty



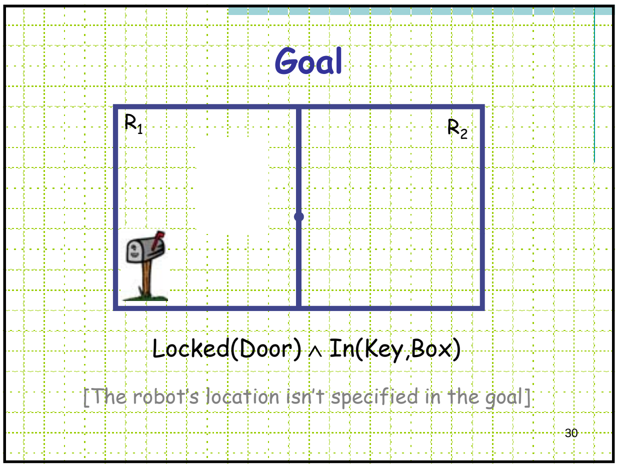
Key-in-Box Example

- The robot must lock the door and put the key in the box
- The key is needed to lock and unlock the door
- Once the key is in the box, the robot can't get it back



Initial State

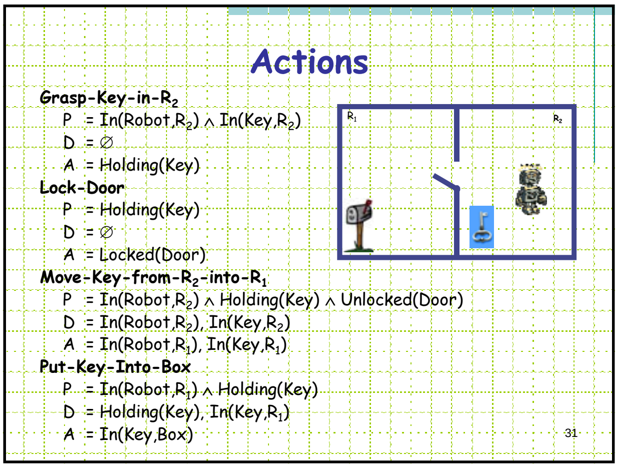
$\mathrm{In}(\mathrm{Robot}, R_2) \wedge \mathrm{In}(\mathrm{Key}, R_2) \wedge \mathrm{Unlocked}(\mathrm{Door})$



Goal

$\mathrm{Locked}(\mathrm{Door}) \wedge \mathrm{In}(\mathrm{Key}, \mathrm{Box})$

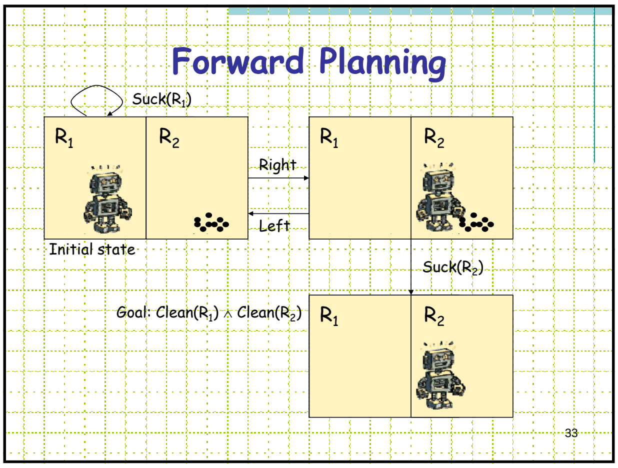
[The robot's location isn't specified in the goal]



Actions

Grasp-Key-in-R_2
P = In(Robot,R_2) ∧ In(Key,R_2)
D = ∅
A = Holding(Key)
Lock-Door
P = Holding(Key)
D = ∅
A = Locked(Door)
Move-Key-from-R_2-into-R_1
P = In(Robot,R_2) ∧ Holding(Key) ∧ Unlocked(Door)
D = In(Robot,R_2), In(Key,R_2)
A = In(Robot,R_1), In(Key,R_1)
Put-Key-Into-Box
P = In(Robot,R_1) ∧ Holding(Key)
D = Holding(Key), In(Key,R_1)
A = In(Key,Box)

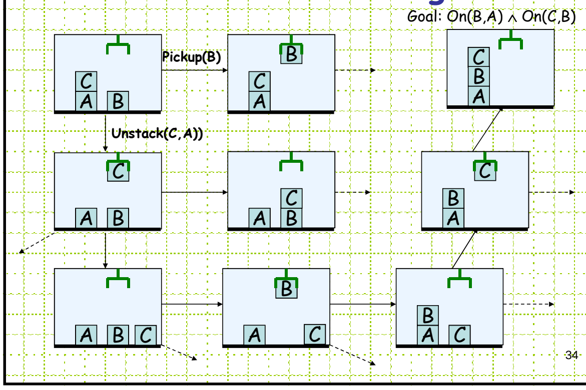
Planning Methods



Forward Planning

Goal: Clean(R_1) ∧ Clean(R_2)

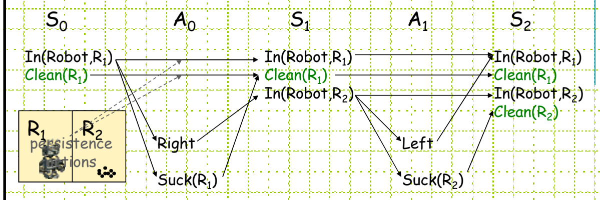
Forward Planning



Need for an Accurate Heuristic

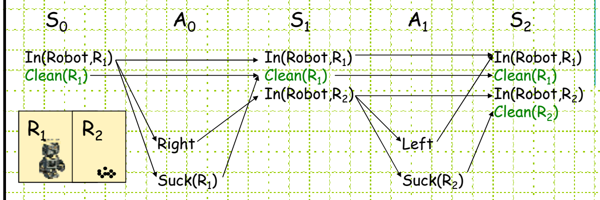
- Forward planning simply searches the space of world states from the initial to the goal state
- Imagine an agent with a large library of actions, whose goal is G, e.g., G = Have(Milk)
- In general, many actions are applicable to any given state, so the branching factor is huge
- In any given state, most applicable actions are irrelevant to reaching the goal Have(Milk)
- Fortunately, an accurate consistent heuristic can be computed using planning graphs

Planning Graph for a State of the Vacuum Robot



- S_0 contains the state's propositions (here, the initial state)
- A_0 contains all actions whose preconditions appear in S_0
- S_1 contains all propositions that were in S_0 or are contained in the add lists of the actions in A_0
- So, S_1 contains all propositions that may be true in the state reached after the first action
- A_1 contains all actions not already in A_0 whose preconditions appear in S_1, hence that may be executable in the state reached after executing the first action. Etc...

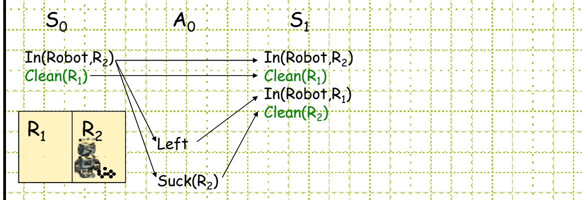
Planning Graph for a State of the Vacuum Robot


S_0 A_0 S_1 A_1 S_2

In(Robot,R_1)
Clean(R_1)
R_1 R_2
Right
Suck(R_1)
In(Robot,R_1)
Clean(R_1)
In(Robot,R_2)
Left
Suck(R_2)
In(Robot,R_1)
Clean(R_1)
In(Robot,R_2)
Clean(R_2)

- The value of i such that S_i contains all the goal propositions is called the level cost of the goal (here i=2)
- By construction of the planning graph, it is a lower bound on the number of actions needed to reach the goal
- In this case, 2 is the actual length of the shortest path to the goal

Planning Graph for Another State

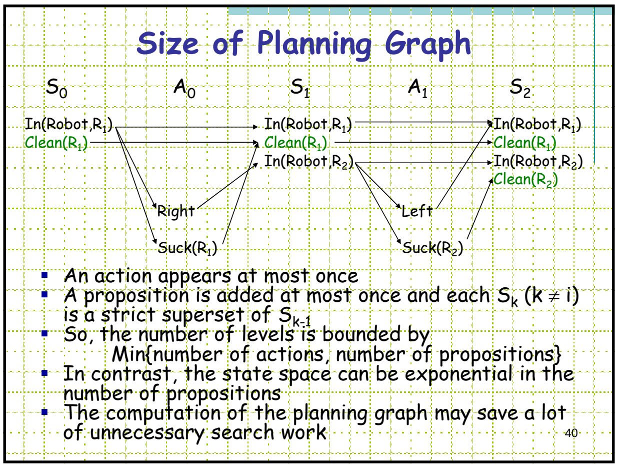

S_0 A_0 S_1

In(Robot,R_2)
Clean(R_1)
R_1 R_2
Left
Suck(R_2)
In(Robot,R_2)
Clean(R_1)
In(Robot,R_1)
Clean(R_2)

- The level cost of the goal is 1, which again is the actual length of the shortest path to the goal

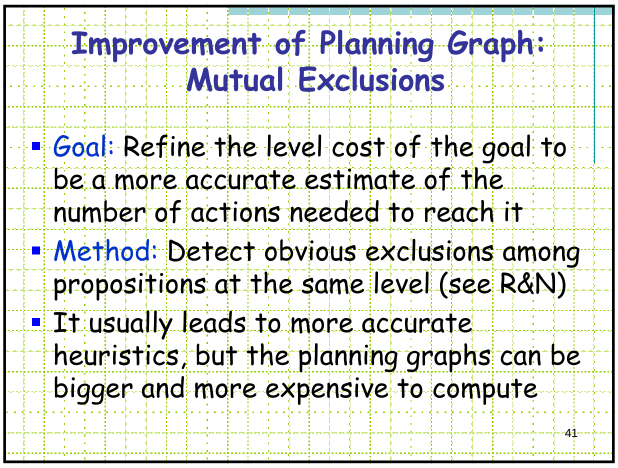
Application of Planning Graphs to Forward Planning

- Whenever a new node is generated, compute the planning graph of its state [update the planning graph at the parent node]
- Stop computing the planning graph when:
    - Either the goal propositions are in a set S_i [then i is the level cost of the goal]
    - Or when $S_{i+1} = S_i$ [then the generated node is **not** on a solution path]
- Set the heuristic h(N) of a node N to the level cost of the goal for the state of N
- h is a consistent heuristic for unit-cost actions
- Hence, A* using h yields a solution with minimum number of actions



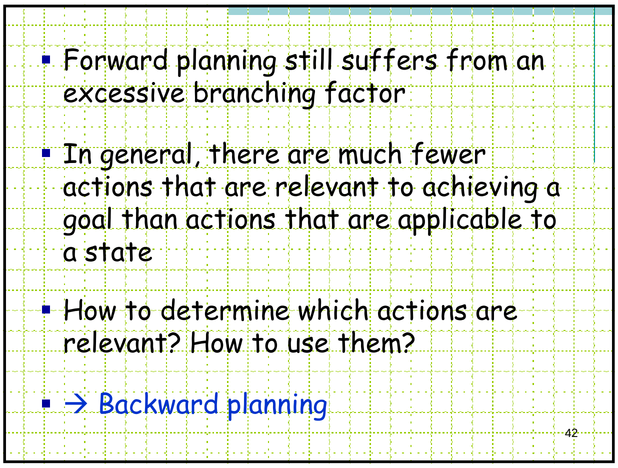
Size of Planning Graph

- An action appears at most once
- A proposition is added at most once and each S_k ($k \neq i$) is a strict superset of S_{k-1}
- So, the number of levels is bounded by
  Min{number of actions, number of propositions}
- In contrast, the state space can be exponential in the number of propositions
- The computation of the planning graph may save a lot of unnecessary search work

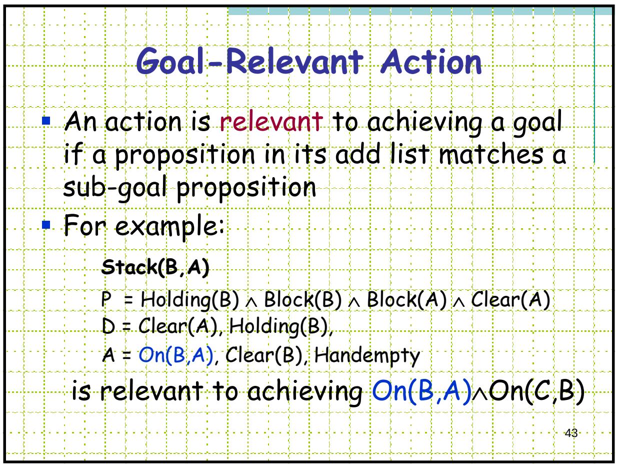


Improvement of Planning Graph: Mutual Exclusions

- Goal: Refine the level cost of the goal to be a more accurate estimate of the number of actions needed to reach it
- Method: Detect obvious exclusions among propositions at the same level (see R&N)
- It usually leads to more accurate heuristics, but the planning graphs can be bigger and more expensive to compute



- Forward planning still suffers from an excessive branching factor
- In general, there are much fewer actions that are relevant to achieving a goal than actions that are applicable to a state
- How to determine which actions are relevant? How to use them?
- → Backward planning

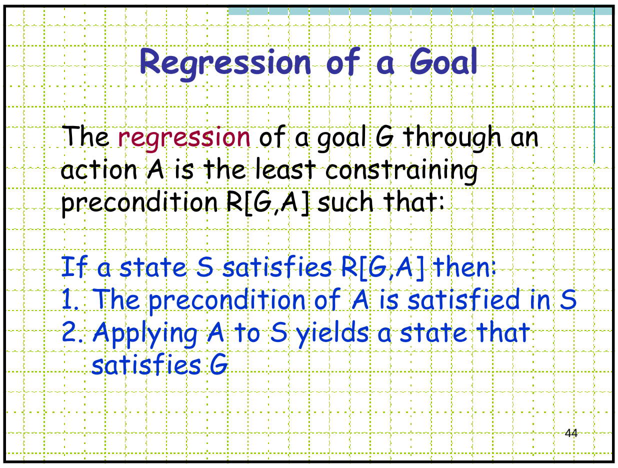


Goal-Relevant Action

- An action is relevant to achieving a goal if a proposition in its add list matches a sub-goal proposition
- For example:

Stack(B,A)
P = Holding(B) ∧ Block(B) ∧ Block(A) ∧ Clear(A)
D = Clear(A), Holding(B),
A = On(B,A), Clear(B), Handempty

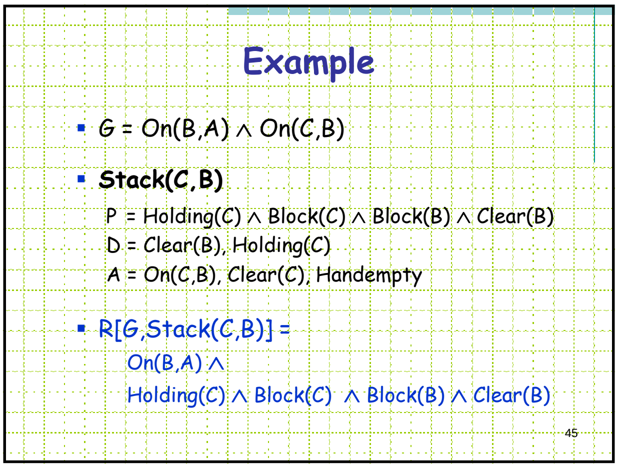
is relevant to achieving On(B,A)∧On(C,B)



Regression of a Goal

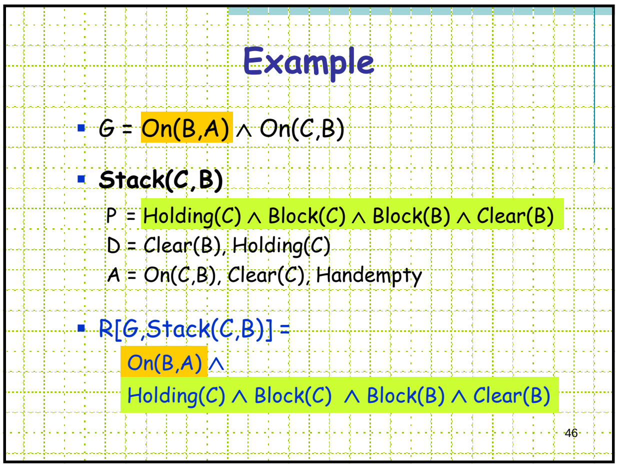
The regression of a goal G through an action A is the least constraining precondition R[G,A] such that:

If a state S satisfies R[G,A] then:
1. The precondition of A is satisfied in S
2. Applying A to S yields a state that satisfies G



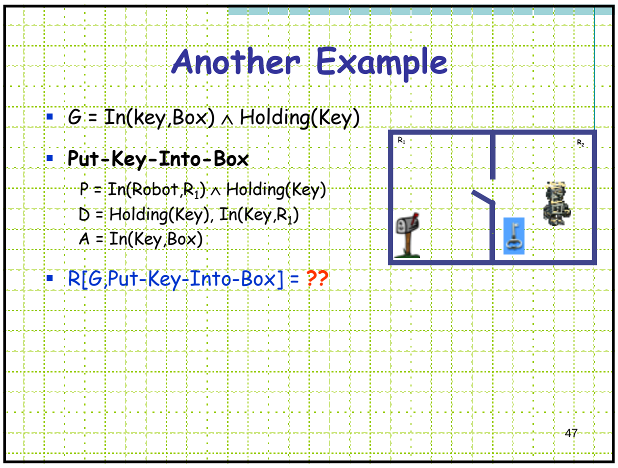
Example

- G = On(B,A) ∧ On(C,B)
- **Stack(C,B)**
  P = Holding(C) ∧ Block(C) ∧ Block(B) ∧ Clear(B)
  D = Clear(B), Holding(C)
  A = On(C,B), Clear(C), Handempty
- R[G,Stack(C,B)] =
  On(B,A) ∧
  Holding(C) ∧ Block(C) ∧ Block(B) ∧ Clear(B)



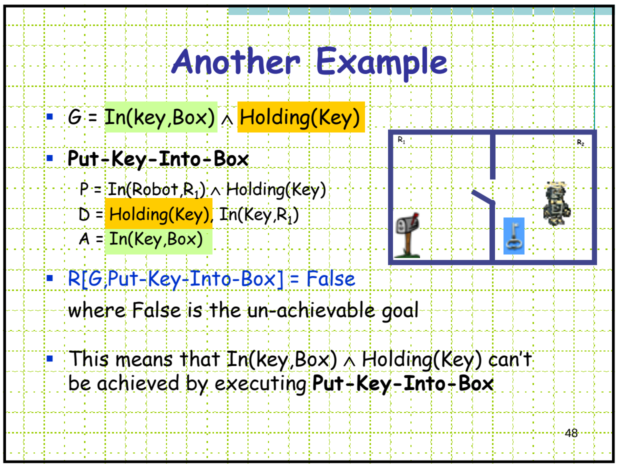
Example

- G = On(B,A) $\wedge$ On(C,B)
- **Stack(C,B)**
  P = Holding(C) $\wedge$ Block(C) $\wedge$ Block(B) $\wedge$ Clear(B)
  D = Clear(B), Holding(C)
  A = On(C,B), Clear(C), Handempty
- R[G,Stack(C,B)] =
  On(B,A) $\wedge$
  Holding(C) $\wedge$ Block(C) $\wedge$ Block(B) $\wedge$ Clear(B)



Another Example

- G = In(key,Box) $\wedge$ Holding(Key)
- **Put-Key-Into-Box**
  P = In(Robot,R_1) $\wedge$ Holding(Key)
  D = Holding(Key), In(Key,R_1)
  A = In(Key,Box)
- R[G,Put-Key-Into-Box] = ??



Another Example

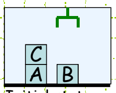
- G = In(key,Box) $\wedge$ Holding(Key)
- **Put-Key-Into-Box**
  P = In(Robot,R_1) $\wedge$ Holding(Key)
  D = Holding(Key), In(Key,R_1)
  A = In(Key,Box)
- R[G,Put-Key-Into-Box] = False
  where False is the un-achievable goal
- This means that In(key,Box) $\wedge$ Holding(Key) can't be achieved by executing **Put-Key-Into-Box**

Computation of R[G,A]

```
1. If any sub-goal of G is in A's delete list
   then return False
2. Else
   a. G' ← Precondition of A
   b. For every sub-goal SG of G do
         If SG is not in A's add list then add SG to G'
3. Return G'
```

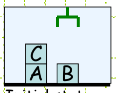
Backward Planning

On(B,A) ∧ On(C,B)



Initial state

Backward Planning



Initial state

On(B,A) ∧ On(C,B)

↓ **Stack(C,B)**

On(B,A) ∧ Holding(C) ∧ Clear(B)

↓ **Pickup(C)**

On(B,A) ∧ Clear(B) ∧ Handempty ∧ Clear(C) ∧ On(C,Table)

↓ **Stack(B,A)**

Clear(C) ∧ On(C,TABLE) ∧ Holding(B) ∧ Clear(A)

↓ **Pickup(B)**

Clear(C) ∧ On(C,Table) ∧ Clear(A) ∧ Handempty ∧ Clear(B) ∧ On(B,Table)

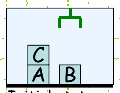
↓ **Putdown(C)**

Clear(A) ∧ Clear(B) ∧ On(B,Table) ∧ Holding(C)

↓ **Unstack(C,A)**

Clear(B) ∧ On(B,Table) ∧ Clear(C) ∧ Handempty ∧ On(C,A)

Backward Planning



Initial state

On(B,A) ∧ On(C,B)

↓ **Stack(C,B)**

On(B,A) ∧ Holding(C) ∧ Clear(B)

↓ **Pickup(C)**

On(B,A) ∧ Clear(B) ∧ Handempty ∧ Clear(C) ∧ On(C,Table)

↓ **Stack(B,A)**

Clear(C) ∧ On(C,TABLE) ∧ Holding(B) ∧ Clear(A)

↓ **Pickup(B)**

Clear(C) ∧ On(C,Table) ∧ Clear(A) ∧ Handempty ∧ Clear(B) ∧ On(B,Table)

↓ **Putdown(C)**

Clear(A) ∧ Clear(B) ∧ On(B,Table) ∧ Holding(C)

↓ **Unstack(C,A)**

Clear(B) ∧ On(B,Table) ∧ Clear(C) ∧ Handempty ∧ On(C,A)

Search Tree

- Backward planning searches a space of goals from the original goal of the problem to a goal that is satisfied in the initial state
- There are often much fewer actions relevant to a goal than there are actions applicable to a state → smaller branching factor than in forward planning
- The lengths of the solution paths are the same

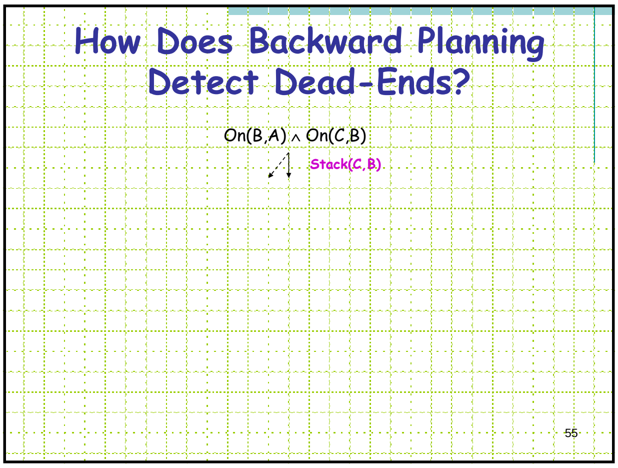
Consistent Heuristic for Backward Planning

A consistent heuristic is obtained as follows :

1. Pre-compute the planning graph of the initial state until it levels off
2. For each node N added to the search tree, set h(N) to the level cost of the goal associated with N

If the goal associated with N can't be satisfied in any set S_k of the planning graph, it can't be achieved, so prune it!

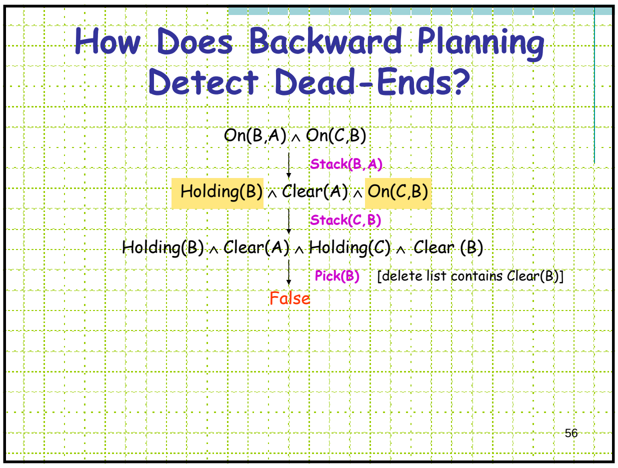
A single planning graph is computed



How Does Backward Planning Detect Dead-Ends?

$\text{On}(B,A) \wedge \text{On}(C,B)$

Stack(C,B)



How Does Backward Planning Detect Dead-Ends?

$\text{On}(B,A) \wedge \text{On}(C,B)$

Stack(B,A)

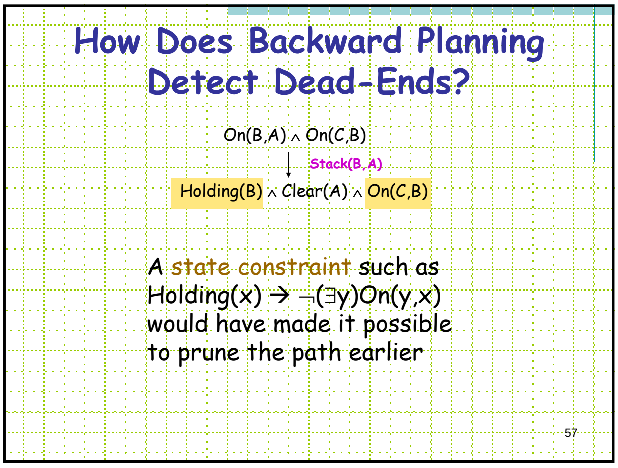
$\text{Holding}(B) \wedge \text{Clear}(A) \wedge \text{On}(C,B)$

Stack(C,B)

$\text{Holding}(B) \wedge \text{Clear}(A) \wedge \text{Holding}(C) \wedge \text{Clear}(B)$

Pick(B) [delete list contains Clear(B)]

False



How Does Backward Planning Detect Dead-Ends?

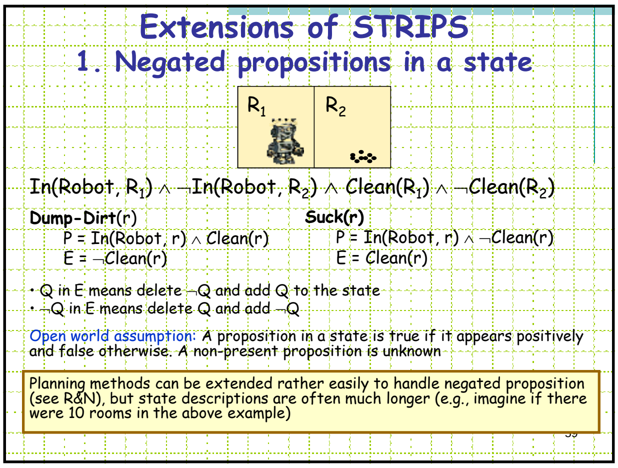
$\text{On}(B,A) \wedge \text{On}(C,B)$

Stack(B,A)

$\text{Holding}(B) \wedge \text{Clear}(A) \wedge \text{On}(C,B)$

A state constraint such as $\text{Holding}(x) \rightarrow \neg(\exists y)\text{On}(y,x)$ would have made it possible to prune the path earlier

Some Extensions of STRIPS Language



Extensions of STRIPS

1. Negated propositions in a state

In(Robot, R_1) ∧ ¬In(Robot, R_2) ∧ Clean(R_1) ∧ ¬Clean(R_2)

Dump-Dirt(r)
P = In(Robot, r) ∧ Clean(r)
E = ¬Clean(r)

Suck(r)
P = In(Robot, r) ∧ ¬Clean(r)
E = Clean(r)

- Q in E means delete ¬Q and add Q to the state
- ¬Q in E means delete Q and add ¬Q

Open world assumption: A proposition in a state is true if it appears positively and false otherwise. A non-present proposition is unknown

Planning methods can be extended rather easily to handle negated proposition (see R&N), but state descriptions are often much longer (e.g., imagine if there were 10 rooms in the above example)

Extensions of STRIPS

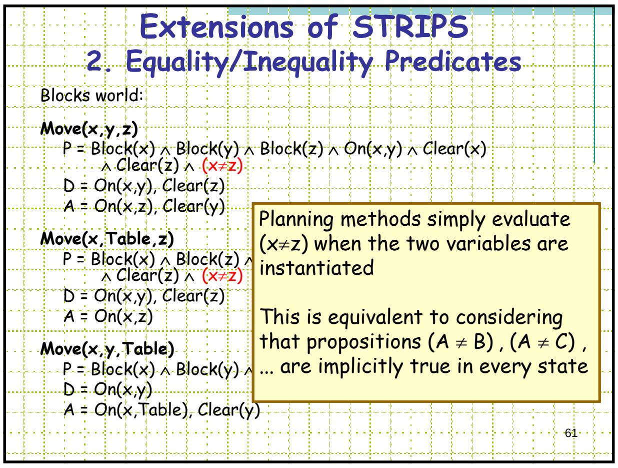
2. Equality/Inequality Predicates

Blocks world:

Move(x,y,z)
P = Block(x) ∧ Block(y) ∧ Block(z) ∧ On(x,y) ∧ Clear(x)
∧ Clear(z) ∧ (x≠z)
D = On(x,y), Clear(z)
A = On(x,z), Clear(y)

Move(x,Table,z)
P = Block(x) ∧ Block(z) ∧ On(x,Table) ∧ Clear(x)
∧ Clear(z) ∧ (x≠z)
D = On(x,y), Clear(z)
A = On(x,z)

Move(x,y,Table)
P = Block(x) ∧ Block(y) ∧ On(x,y) ∧ Clear(x)
D = On(x,y)
A = On(x,Table), Clear(y)



Extensions of STRIPS

2. Equality/Inequality Predicates

Blocks world:

Move(x,y,z)

$P = Block(x) \wedge Block(y) \wedge Block(z) \wedge On(x,y) \wedge Clear(x)$
$\wedge\ Clear(z) \wedge (x \neq z)$
$D = On(x,y), Clear(z)$
$A = On(x,z), Clear(y)$

Move(x,Table,z)

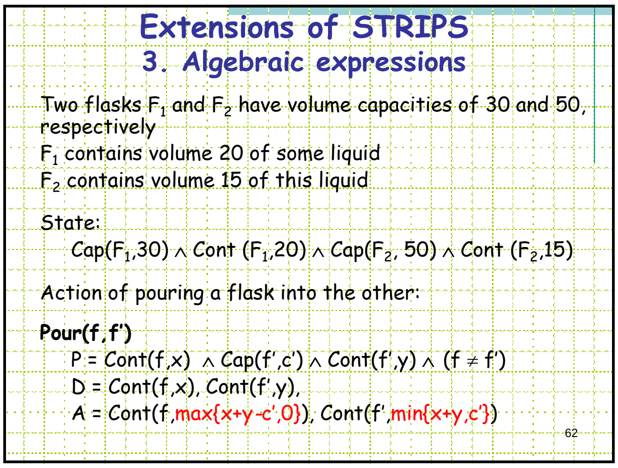
$P = Block(x) \wedge Block(z) \wedge$
$\wedge\ Clear(z) \wedge (x \neq z)$
$D = On(x,y), Clear(z)$
$A = On(x,z)$

Move(x,y,Table)

$P = Block(x) \wedge Block(y) \wedge$
$D = On(x,y)$
$A = On(x,Table), Clear(y)$

Planning methods simply evaluate ($x \neq z$) when the two variables are instantiated

This is equivalent to considering that propositions ($A \neq B$) , ($A \neq C$) , ... are implicitly true in every state



Extensions of STRIPS

3. Algebraic expressions

Two flasks F_1 and F_2 have volume capacities of 30 and 50, respectively

F_1 contains volume 20 of some liquid

F_2 contains volume 15 of this liquid

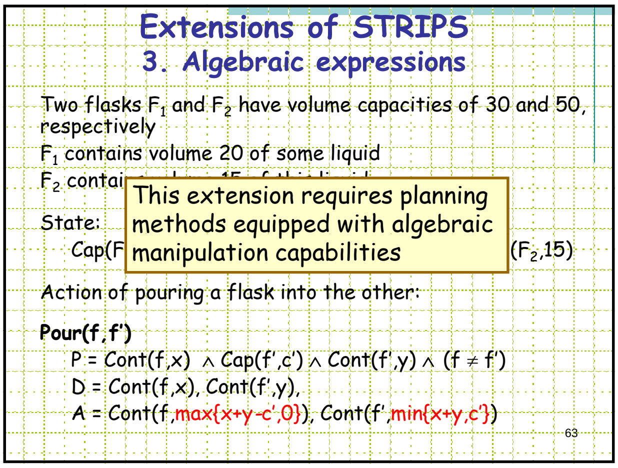
State:

$Cap(F_1,30) \wedge Cont(F_1,20) \wedge Cap(F_2, 50) \wedge Cont(F_2,15)$

Action of pouring a flask into the other:

Pour(f,f')

$P = Cont(f,x) \wedge Cap(f',c') \wedge Cont(f',y) \wedge (f \neq f')$
$D = Cont(f,x), Cont(f',y),$
$A = Cont(f,\max\{x+y-c',0\}), Cont(f',\min\{x+y,c'\})$



Extensions of STRIPS

3. Algebraic expressions

Two flasks F_1 and F_2 have volume capacities of 30 and 50, respectively

F_1 contains volume 20 of some liquid

F_2 contai

State:

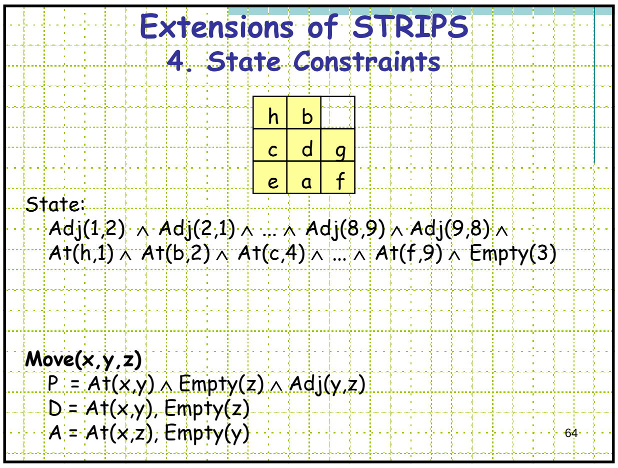
Cap(F $(F_2,15)$

This extension requires planning methods equipped with algebraic manipulation capabilities

Action of pouring a flask into the other:

Pour(f,f')

$P = Cont(f,x) \wedge Cap(f',c') \wedge Cont(f',y) \wedge (f \neq f')$
$D = Cont(f,x), Cont(f',y),$
$A = Cont(f,\max\{x+y-c',0\}), Cont(f',\min\{x+y,c'\})$



Extensions of STRIPS
4. State Constraints

h	b	
c	d	g
e	a	f

State:
Adj(1,2) ∧ Adj(2,1) ∧ … ∧ Adj(8,9) ∧ Adj(9,8) ∧
At(h,1) ∧ At(b,2) ∧ At(c,4) ∧ … ∧ At(f,9) ∧ Empty(3)

Move(x,y,z)
P = At(x,y) ∧ Empty(z) ∧ Adj(y,z)
D = At(x,y), Empty(z)
A = At(x,z), Empty(y)

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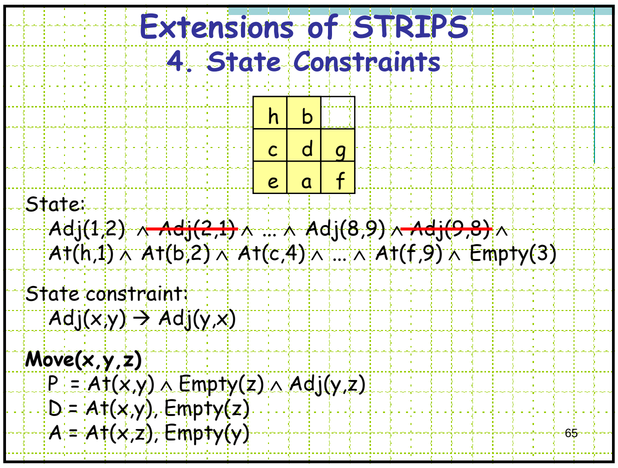

Extensions of STRIPS
4. State Constraints

h	b	
c	d	g
e	a	f

State:
Adj(1,2) ∧ ~~Adj(2,1)~~ ∧ … ∧ Adj(8,9) ∧ ~~Adj(9,8)~~ ∧
At(h,1) ∧ At(b,2) ∧ At(c,4) ∧ … ∧ At(f,9) ∧ Empty(3)

State constraint:
Adj(x,y) → Adj(y,x)

Move(x,y,z)
P = At(x,y) ∧ Empty(z) ∧ Adj(y,z)
D = At(x,y), Empty(z)
A = At(x,z), Empty(y)

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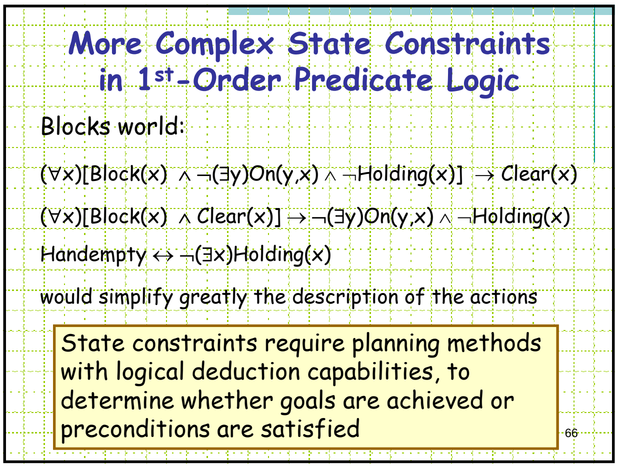

More Complex State Constraints in 1st-Order Predicate Logic

Blocks world:

(∀x)[Block(x) ∧ ¬(∃y)On(y,x) ∧ ¬Holding(x)] → Clear(x)

(∀x)[Block(x) ∧ Clear(x)] → ¬(∃y)On(y,x) ∧ ¬Holding(x)

Handempty ↔ ¬(∃x)Holding(x)

would simplify greatly the description of the actions

State constraints require planning methods with logical deduction capabilities, to determine whether goals are achieved or preconditions are satisfied

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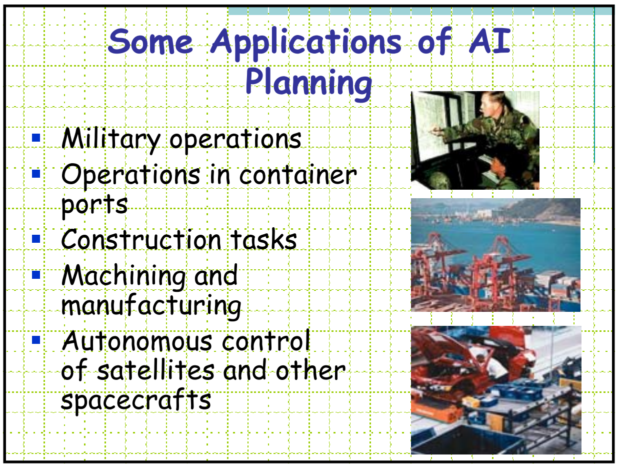

Some Applications of AI Planning

- Military operations
- Operations in container ports
- Construction tasks
- Machining and manufacturing
- Autonomous control of satellites and other spacecrafts

Deep Space One

Started: January 1996
Launch: October 15th, 1998
http://ic.arc.nasa.gov/projects/remote-agent/pstext.html