Blind (Uninformed) **Search**

(Where we systematically explore alternatives)

R&N: Chap. 3, Sect. 3.3–5

Slides from Jean-Claude Latombe at Stanford University
(used with permission)

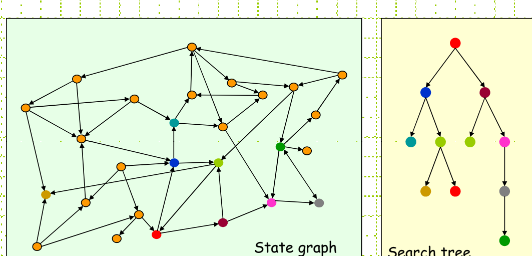
1

Simple Problem-Solving-Agent Agent Algorithm

1. $s_0 \leftarrow$ sense/read initial state
2. GOAL? $\leftarrow$ select/read goal test
3. Succ $\leftarrow$ read successor function
4. solution $\leftarrow$ **search**(s_0, GOAL?, Succ)
5. perform(solution)

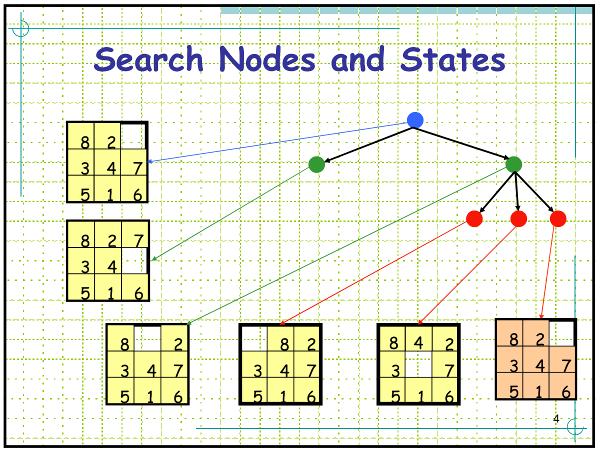
2

Search Tree



Note that some states may be visited multiple times

3


Search Nodes and States
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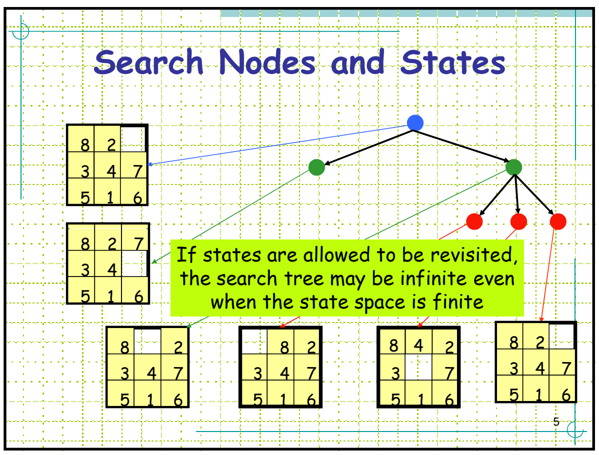
Search Nodes and States
If states are allowed to be revisited, the search tree may be infinite even when the state space is finite
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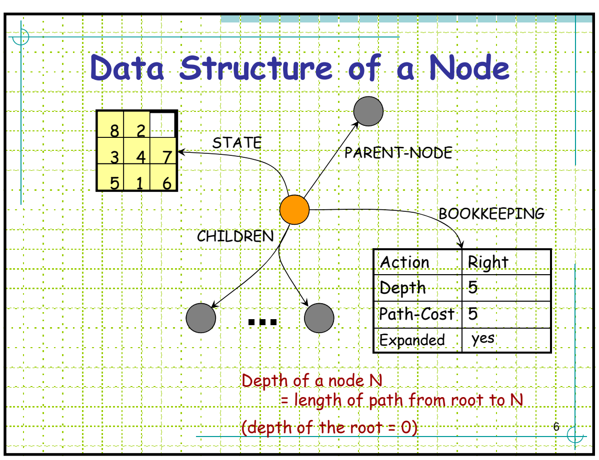
Data Structure of a Node
STATE
PARENT-NODE
BOOKKEEPING
CHILDREN
Action Right
Depth 5
Path-Cost 5
Expanded yes
Depth of a node N
= length of path from root to N
(depth of the root = 0)
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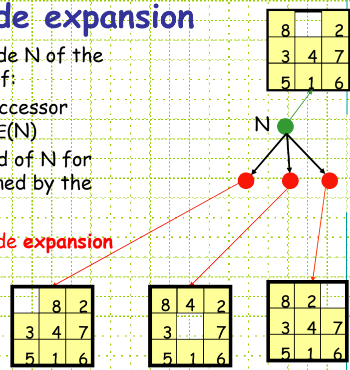
Node expansion

The expansion of a node N of the search tree consists of:

1) Evaluating the successor function on STATE(N)
2) Generating a child of N for each state returned by the function

node **generation** ≠ node **expansion**



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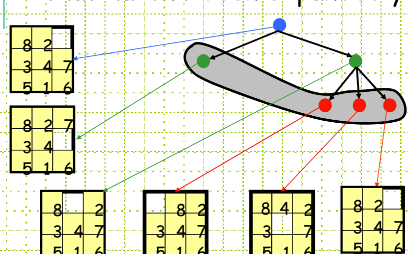
Fringe of Search Tree

- The fringe is the set of all search nodes that haven't been expanded yet



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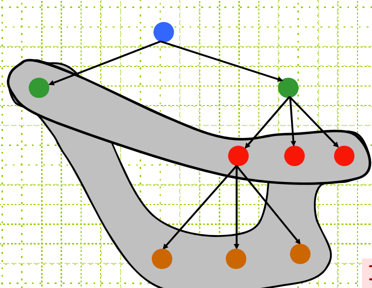

Is it identical to the set of leaves?

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Search Strategy

- The fringe is the set of all search nodes that haven't been expanded yet
- The fringe is implemented as a priority queue FRINGE
  - INSERT(node,FRINGE)
  - REMOVE(FRINGE)
- The ordering of the nodes in FRINGE defines the search strategy

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Search Algorithm #1

SEARCH#1

1. If GOAL?(initial-state) then return initial-state
2. INSERT(initial-node,FRINGE)
3. Repeat:
   a. If empty(FRINGE) then return failure
   b. N ← REMOVE(FRINGE)   Expansion of N
   c. s ← STATE(N)
   d. For every state s' in SUCCESSORS(s)
      i. Create a new node N' as a child of N
      ii. If GOAL?(s') then return path or goal state
      iii. INSERT(N',FRINGE)

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Performance Measures

- Completeness
  A search algorithm is complete if it finds a solution whenever one exists
  [What about the case when no solution exists?]
- Optimality
  A search algorithm is optimal if it returns a minimum-cost path whenever a solution exists
- Complexity
  It measures the time and amount of memory required by the algorithm

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Blind vs. Heuristic Strategies

- Blind (or un-informed) strategies do not exploit state descriptions to order FRINGE. They only exploit the positions of the nodes in the search tree
- Heuristic (or informed) strategies exploit state descriptions to order FRINGE (the most "promising" nodes are placed at the beginning of FRINGE)

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Example

For a blind strategy, N_1 and N_2 are just two nodes (at some position in the search tree)

STATE N_1

8	2	
3	4	7
5	1	6

STATE N_2

1	2	3
4	5	
7	8	6

1	2	3
4	5	6
7	8	

Goal state

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Example

For a heuristic strategy counting the number of misplaced tiles, N_2 is more promising than N_1

STATE N_1

8	2	
3	4	7
5	1	6

STATE N_2

1	2	3
4	5	
7	8	6

1	2	3
4	5	6
7	8	

Goal state

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Remark

- Some search problems, such as the (n^2-1)-puzzle, are NP-hard
- One can't expect to solve all instances of such problems in less than exponential time (in n)
- One may still strive to solve each instance as efficiently as possible

→ This is the purpose of the search strategy

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Blind Strategies

- Breadth-first
  - Bidirectional
- Depth-first
  - Depth-limited
  - Iterative deepening

Arc cost = 1

- Uniform-Cost (variant of breadth-first)

Arc cost = c(action) $\geq \varepsilon > 0$

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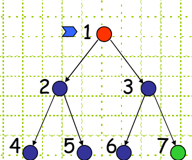
Breadth-First Strategy

New nodes are inserted at the end of FRINGE



FRINGE = (1)

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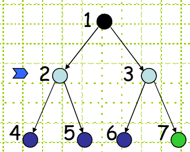
Breadth-First Strategy

New nodes are inserted at the end of FRINGE



FRINGE = (2, 3)

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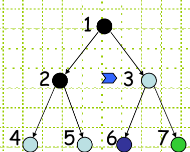
Breadth-First Strategy

New nodes are inserted at the end of FRINGE



FRINGE = (3, 4, 5)

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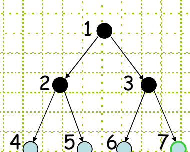
Breadth-First Strategy

New nodes are inserted at the end of FRINGE



FRINGE = (4, 5, 6, 7)

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Important Parameters

1) Maximum number of successors of any state

   → branching factor b of the search tree

2) Minimal length (≠ cost) of a path between the initial and a goal state

   → depth d of the shallowest goal node in the search tree

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete? Not complete?
  - Optimal? Not optimal?

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:
  ???

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:
  $1 + b + b^2 + \ldots + b^d = ???$

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Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:
  $1 + b + b^2 + \ldots + b^d = O(b^d)$
- → Time and space complexity is $O(b^d)$

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Big O Notation

$g(n) = O(f(n))$ if there exist two positive constants a and N such that:

for all $n > N$: $g(n) \le a \times f(n)$

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Time and Memory Requirements

d	# Nodes	Time	Memory
2	111	.01 msec	11 Kbytes
4	11,111	1 msec	1 Mbyte
6	~10^6	1 sec	100 Mb
8	~10^8	100 sec	10 Gbytes
10	~10^{10}	2.8 hours	1 Tbyte
12	~10^{12}	11.6 days	100 Tbytes
14	~10^{14}	3.2 years	10,000 Tbytes

Assumptions: b = 10; 1,000,000 nodes/sec; 100bytes/node

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Time and Memory Requirements

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14	~10^{14}	3.2 years	10,000 Tbytes

Assumptions: b = 10; 1,000,000 nodes/sec; 100bytes/node

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Remark

If a problem has no solution, breadth-first may run for ever (if the state space is infinite or states can be revisited arbitrary many times)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

?

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

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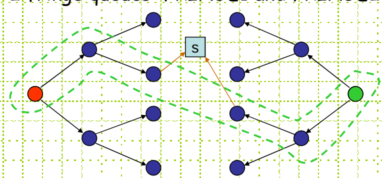
Bidirectional Strategy

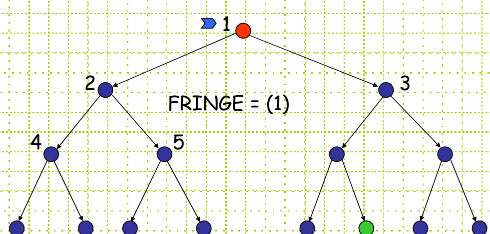
2 fringe queues: FRINGE1 and FRINGE2



Time and space complexity is $O(b^{d/2}) << O(b^d)$ if both trees have the same branching factor b

Depth-First Strategy

New nodes are inserted at the front of FRINGE



Depth-First Strategy

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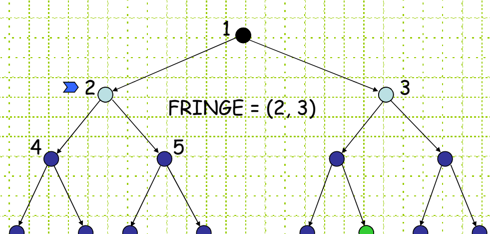

Depth-First Strategy

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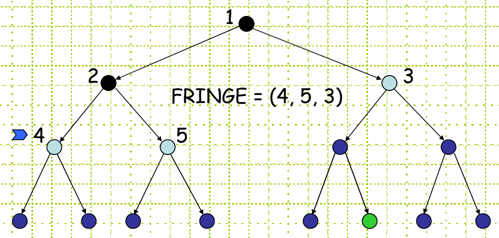

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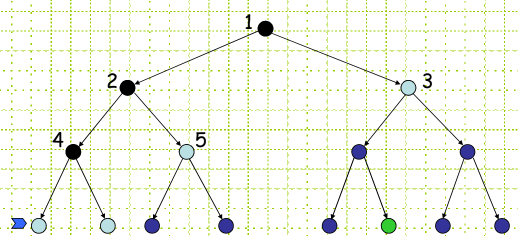
Depth-First Strategy

New nodes are inserted at the front of FRINGE



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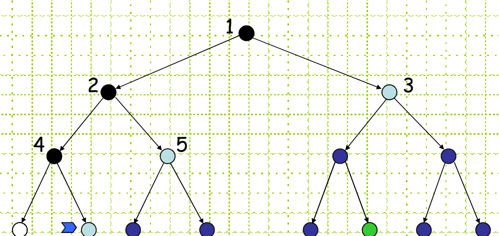
Depth-First Strategy

New nodes are inserted at the front of FRINGE



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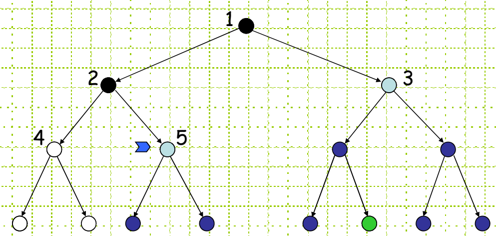
Depth-First Strategy

New nodes are inserted at the front of FRINGE



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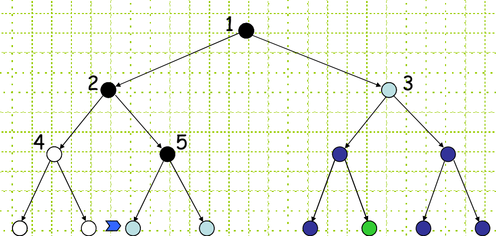
Depth-First Strategy

New nodes are inserted at the front of FRINGE



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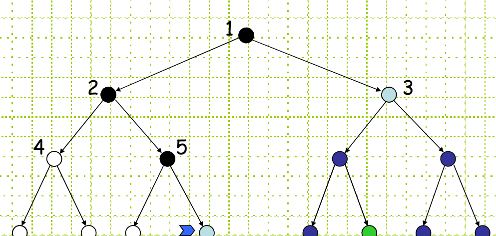
Depth-First Strategy

New nodes are inserted at the front of FRINGE



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Depth-First Strategy

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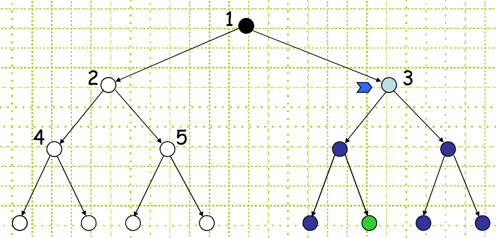

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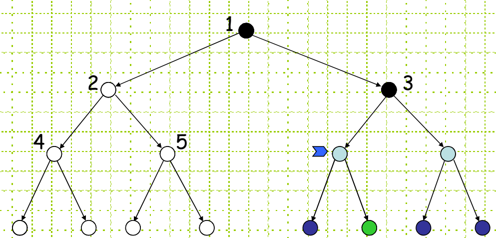
Depth-First Strategy

New nodes are inserted at the front of FRINGE



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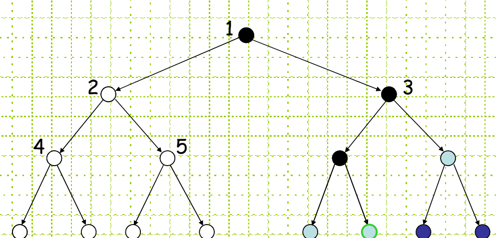
Depth-First Strategy

New nodes are inserted at the front of FRINGE



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Evaluation

- b: branching factor
- d: depth of shallowest goal node
- m: maximal depth of a leaf node
- Depth-first search is:
  - Complete?
  - Optimal?

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- m: maximal depth of a leaf node
- Depth-first search is:
  - Complete only for finite search tree
  - Not optimal
- Number of nodes generated (worst case): $1 + b + b^2 + \ldots + b^m = O(b^m)$
- Time complexity is $O(b^m)$
- Space complexity is $O(bm)$ [or $O(m)$]

[Reminder: Breadth-first requires $O(b^d)$ time and space]

Depth-Limited Search

- Depth-first with depth cutoff k (depth at which nodes are not expanded)
- Three possible outcomes:
  - Solution
  - Failure (no solution)
  - Cutoff (no solution within cutoff)

Iterative Deepening Search

Provides the best of both breadth-first and depth-first search

IDS
For k = 0, 1, 2, … do:
    Perform depth-first search with depth cutoff k
    (i.e., only generate nodes with depth ≤ k)

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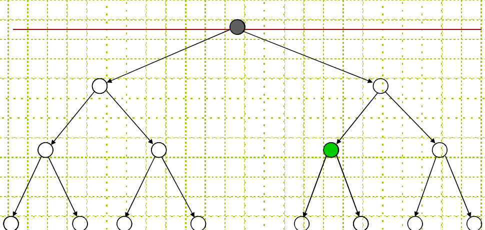
Iterative Deepening



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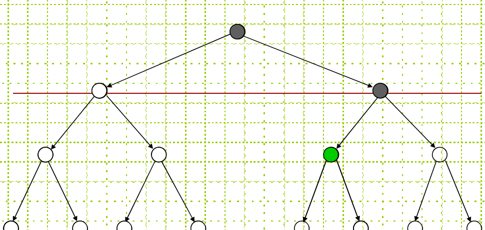
Iterative Deepening



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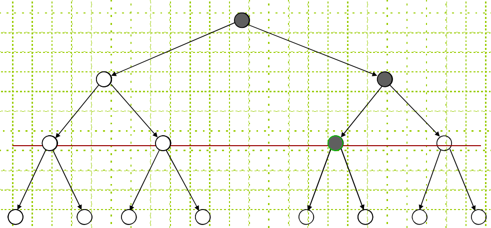
Iterative Deepening



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Performance

- Iterative deepening search is:
  - Complete
  - Optimal if step cost =1
- Time complexity is:
  $(d+1)(1) + db + (d-1)b^2 + \ldots + (1)\, b^d = O(b^d)$
- Space complexity is: $O(bd)$ or $O(d)$

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Calculation

$$db + (d-1)b^2 + \ldots + (1)\, b^d$$
$$= b^d + 2b^{d-1} + 3b^{d-2} + \ldots + db$$
$$= (1 + 2b^{-1} + 3b^{-2} + \ldots + db^{-d}) \times b^d$$
$$\leq \left(\Sigma_{i=1,\ldots,\infty}\, ib^{(1-i)}\right) \times b^d \;=\; b^d\,(b/(b-1))^2$$

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Number of Generated Nodes
(Breadth-First & Iterative Deepening)

d = 5 and b = 2

BF	ID
1	1 x 6 = 6
2	2 x 5 = 10
4	4 x 4 = 16
8	8 x 3 = 24
16	16 x 2 = 32
32	32 x 1 = 32
63	120

120/63 ~ 2

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Number of Generated Nodes
(Breadth-First & Iterative Deepening)

d = 5 and b = 10

BF	ID
1	6
10	50
100	400
1,000	3,000
10,000	20,000
100,000	100,000
111,111	123,456

123,456/111,111 ~ 1.111

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Comparison of Strategies

- Breadth-first is complete and optimal, but has high space complexity
- Depth-first is space efficient, but is neither complete, nor optimal
- Iterative deepening is complete and optimal, with the same space complexity as depth-first and almost the same time complexity as breadth-first

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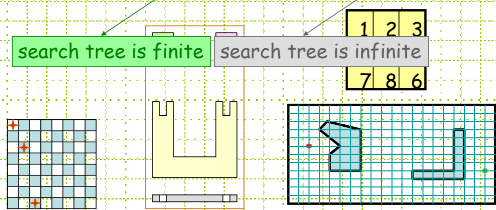
Revisited States

No Few Many



8-queens

assembly planning

8-puzzle and robot navigation

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Avoiding Revisited States

- Requires comparing state descriptions
- Breadth-first search:
  - Store all states associated with **generated** nodes in VISITED
  - If the state of a new node is in VISITED, then discard the node

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Avoiding Revisited States

- Requires comparing state descriptions
- Breadth-first search:
  - Store all states associated with **generated** nodes in VISITED
  - If the state of a new node is in VISITED, then discard the node

Implemented as hash-table
or as explicit data structure with flags

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Avoiding Revisited States

- Depth-first search:

  Solution 1:
  - Store all states associated with nodes in current path in VISITED
  - If the state of a new node is in VISITED, then discard the node

  → ??

Avoiding Revisited States

- Depth-first search:

  Solution 1:
  - Store all states associated with nodes in current path in VISITED
  - If the state of a new node is in VISITED, then discard the node

  → Only avoids loops

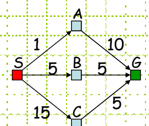
  Solution 2:
  - Store all generated states in VISITED
  - If the state of a new node is in VISITED, then discard the node

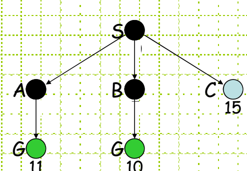
  → Same space complexity as breadth-first !

Uniform-Cost Search

- Each arc has some cost $c \geq \varepsilon > 0$
- The cost of the path to each node N is

  $g(N) = \Sigma$ costs of arcs
- The goal is to generate a solution path of minimal cost
- The nodes N in the queue FRINGE are sorted in increasing g(N)





- Need to modify search algorithm

Search Algorithm #2

SEARCH#2
1. INSERT(initial-node,FRINGE)
2. Repeat:
   a. If empty(FRINGE) then return failure
   b. N ← REMOVE(FRINGE)
   c. s ← STATE(N)
   d. If GOAL?(s) then return path or goal state
   e. For every state s' in SUCCESSORS(s)
      i. Create a node N' as a successor of N
      ii. INSERT(N',FRINGE)

The goal test is applied to a node when this node is **expanded**, not when it is generated.

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Avoiding Revisited States in Uniform-Cost Search

- For any state S, when the first node N such that STATE(N) = S is expanded, the path to N is the best path from the initial state to S
- So:
  - When a node is **expanded**, store its state into CLOSED
  - When a new node N is generated:
    - If STATE(N) is in CLOSED, discard N
    - If there exits a node N' in the fringe such that STATE(N') = STATE(N), discard the node — N or N' — with the highest-cost path

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