First-Order Logic

Chapter 8

Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Propositional logic, pros and cons

- ☺ Propositional logic is declarative
- ☺ Propositional logic allows partial (disjunctive/negated) information
  - (unlike most data structures and databases)
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

Propositional logic, pros and cons

- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)
- ☹ Propositional logic has very limited expressive power
  - (unlike natural language)
  - E.g., cannot say "pits cause breezes in adjacent squares"
    - except by writing one sentence for each square

Why not use Natural Language?

- It serves a different purpose:
  - Communication rather than representation
- It is not compositional
  - Context matters
- It can be ambiguous
  - Again, context matters

Create a new language

- Builds on propositinal logic
- But is inspired by natural language!

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of FOL: Basic elements

- Constants KingJohn, 2, NUS,...
- Predicates Brother, >,...
- Functions Sqrt, LeftLegOf,...
- Variables x, y, a, b,...
- Connectives $\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
- Equality =
- Quantifiers $\forall, \exists$

Atomic sentences

Atomic sentence = *predicate* (*term*$_1$,...,*term*$_n$) or *term*$_1$ = *term*$_2$

Term = *function* (*term*$_1$,...,*term*$_n$) or *constant* or *variable*

- E.g., *Brother(KingJohn,RichardTheLionheart)*

  >(Length(LeftLegOf(Richard)),Length(LeftLegOf(KingJohn)))

Complex sentences

- Complex sentences are made from atomic sentences using connectives
  $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \Rightarrow S_2$, $S_1 \Leftrightarrow S_2$,

E.g. *Sibling(KingJohn,Richard)* $\Rightarrow$ *Sibling(Richard,KingJohn)*

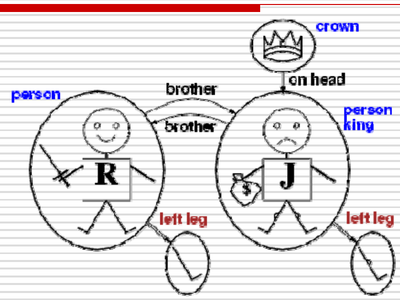
$>(1,2) \vee \leq (1,2)$

$<(1,2) \wedge \neg >(1,2)$

Truth in first-order logic

- Sentences are true with respect to a model and an interpretation (DIAGRAM)
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for
  constant symbols → objects
  predicate symbols → relations
  function symbols → functional relations
- An atomic sentence $predicate(term_1,\ldots,term_n)$ is true iff the objects referred to by $term_1,\ldots,term_n$ are in the relation referred to by *predicate*

Models for FOL: Example



Universal quantification

- ∀*<variables> <sentence>*

Everyone in HR is smart:
$\forall x\ At(x,HR) \Rightarrow Smart(x)$

- ∀x *P* is true in a model *m* iff *P* is true with *x* being each possible object in the model
- Roughly speaking, equivalent to the conjunction of instantiations of *P*

$At(KingJohn,HR) \Rightarrow Smart(KingJohn)$
$\wedge\ At(Richard,HR) \Rightarrow Smart(Richard)$
$\wedge\ At(HR,HR) \Rightarrow Smart(HR)$
$\wedge \ldots$

A common mistake to avoid

- Typically, ⇒ is the main connective with ∀
- Common mistake: using ∧ as the main connective with ∀:

$\forall x\ At(x,HR) \wedge Smart(x)$

means "Everyone is at HR and everyone is smart"

Existential quantification

- ∃*<variables> <sentence>*

Someone at HR is smart:
$\exists x\ At(x,HR) \wedge Smart(x)$

- ∃*x P* is true in a model *m* iff *P* is true with *x* being some possible object in the model
- Roughly speaking, equivalent to the disjunction of instantiations of *P*

$At(KingJohn,HR) \wedge Smart(KingJohn)$
$\vee\ At(Richard,HR) \wedge Smart(Richard)$
$\vee\ At(HR,HR) \wedge Smart(HR)$
$\vee \ldots$

Another mistake to avoid

- Typically, ∧ is the main connective with ∃
- Common mistake: using ⇒ as the main connective with ∃:

  ∃*x* At(x,HR) ⇒ Smart(x)

  is true if there is anyone who is not at HR!

Properties of quantifiers

- ∀x ∀y is the same as ∀y ∀x
- ∃x ∃y is the same as ∃y ∃x
- ∃x ∀y is not the same as ∀y ∃x
- ∃x ∀y Loves(x,y)
  - "There is a person who loves everyone in the world"
- ∀y ∃x Loves(x,y)
  - "Everyone in the world is loved by at least one person"
- Quantifier duality: each can be expressed using the other

  ∀x Likes(x,IceCream) ¬∃x ¬Likes(x,IceCream)

  ∃x Likes(x,Broccoli) ¬∀x ¬Likes(x,Broccoli)

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- E.g., definition of *Sibling* in terms of *Parent*:

  ∀*x,y Sibling(x,y)* ⇔

  [¬(x = y) ∧ ∃m,f ¬ (m = f) ∧ Parent(m,x) ∧ Parent(f,x) ∧ Parent(m,y) ∧ Parent(f,y)]

Using FOL

The kinship domain:

- Brothers are siblings
  $\forall x,y\ Brother(x,y) \Leftrightarrow Sibling(x,y)$
- One's mother is one's female parent
  $\forall m,c\ Mother(c) = m \Leftrightarrow (Female(m) \wedge Parent(m,c))$
- "Sibling" is symmetric
  $\forall x,y\ Sibling(x,y) \Leftrightarrow Sibling(y,x)$

Some sentences are ***Axioms*** *(i.e. definitions, facts) while others are* ***Theorems*** *derived from those.*

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at $t=5$:

  Tell(KB,Percept([Smell,Breeze,None],5))
  Ask(KB,∃a BestAction(a,5))

- I.e., does the KB entail some best action at $t=5$?
- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)
- Given a sentence *S* and a substitution q,
- *S*q denotes the result of plugging q into *S*; e.g.,
  S = Smarter(x,y)
  q = {x/Hillary,y/Bill}
  *S*q = Smarter(Hillary,Bill)
- Ask(KB,S) returns some/all q such that KB ⊨ Sq

KB for the wumpus world

- Perception
  - $\forall t,s,b\ Percept([s,b,Glitter],t) \Rightarrow Glitter(t)$
- Reflex
  - $\forall t\ Glitter(t) \Rightarrow BestAction(Grab,t)$

Deducing hidden properties

- $\forall x,y,a,b\ Adjacent([x,y],[a,b]) \Leftrightarrow [a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$

Properties of squares:

- $\forall s,t\ At(Agent,s,t) \wedge Breeze(t) \Rightarrow Breezy(s)$

Squares are breezy near a pit:

- Diagnostic rule---infer cause from effect
  $\forall s\ Breezy(s) \Rightarrow \exists r\ Adjacent(r,s) \wedge Pit(r)$
- Causal rule---infer effect from cause
  $\forall r\ Pit(r) \Rightarrow [\forall s\ Adjacent(r,s) \Rightarrow Breezy(s)\ ]$

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers
- Increased expressive power: sufficient to define wumpus world